

Reg. No.

Question Paper Code

11666

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022

Fourth Semester

Artificial Intelligence and Data Science

20BSMA404 - LINEAR ALGEBRA AND ITS APPLICATIONS

(Regulations 2020)

Duration: 3 Hours

Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

- |  | <i>Marks,<br/>K-Level, CO</i> |
|--|-------------------------------|
| 1. Define rank of a matrix.  | 2,K1,CO1                      |
| 2. Solve $x + 2y = 1$ and $3x - 2y = 7$ .  | 2,K3,CO1                      |
| 3. What is the dimension of vector space $P_n(\mathbb{R})$ ?   | 2,K2,CO2                      |
| 4. Define basis.   | 2,K1,CO2                      |
| 5. If $T: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $T(x) = 2^x, \forall x \in \mathbb{R}$ , show that $T$ is not linear. | 2,K2,CO3                      |
| 6. State dimension theorem.  | 2,K1,CO3                      |
| 7. Define inner product space.   | 2,K1,CO4                      |
| 8. State triangle inequality.  | 2,K1,CO4                      |
| 9. Define singular value.  | 2,K1,CO5                      |
| 10. Define the term Image Processing.  | 2,K1,CO5                      |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a)(i)

Find the value of a and b if the matrix is  $A = \begin{pmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{pmatrix}$  of

rank 2.

(ii) Find  $\rho(A)$  if  $A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}$

8,K3,CO1

8,K3,CO1

OR

- b) Solve the system of equation  $2x - 6y + 8z = 24$ ,  
 $5x + 4y - 3z = 2$ ,  $3x + y + 2z = 16$  by LU decomposition method. 16, K3, CO1

12. a) Prove that  $F^n$  is a vector space over a field  $F$  under addition and multiplication defined by 16, K3, CO2

$$(x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n) = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n) \text{ and } a(x_1, x_2, \dots, x_n) = (ax_1, ax_2, \dots, ax_n)$$

**OR**

- b) Test whether the polynomial  $x^3 - 3x + 5$  is a linear combination of  $x^3 + 2x^2 - x + 1$  and  $x^3 + 3x^2 - 1$  in  $P_3(R)$ . 16, K3, CO2

13. a) Let  $T : M_{2 \times 2}(R) \rightarrow P_2(R)$  be linear transformation defined by 16, K3, CO3

$$T \begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+b) + 2dx + bx^2. \text{ find } [T]_{\beta}^{\gamma} \text{ where } \beta \text{ \& } \gamma \text{ are the}$$

standard order basis of the matrix of linear transformation with respect to standard order basis of  $M_{2 \times 2}(R)$ ,  $P_2(R)$  respectively.

**OR**

- b) Let  $T : R^3 \rightarrow R^2$  be linear transformation defined by 16, K3, CO3  
 $T(x, y, z) = (x - y, 2z)$ . Find  $N(T)$ ,  $R(T)$ , nullity and rank of  $T$ .

14. a) 16, K3, CO4

Find the QR decomposition for the matrix 
$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

**OR**

- b) If  $V = P(R)$ , the vector space of polynomials over  $R$  with inner product defined by 16, K3, CO4

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt, \text{ where } f(t) = t + 2, g(t) = t^2 - 2t - 3. \text{ find}$$

$$\langle f, g \rangle, \|f\|, \|g\|.$$

15. a) Find the matrix  $U, \Sigma, V$  such that  $A = U \Sigma V^T$ , where  $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$ . 16, K3, CO5

**OR**

- b) Discuss Image Processing with example. 16, K3, CO5