Reg. No.												

Question Paper Code	11668

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022

Fourth Semester

Computer Science and Engineering

(Common to Information Technology)

20BSMA402 – PROBABILITY AND QUEUING THEORY

(Regulations 2020)

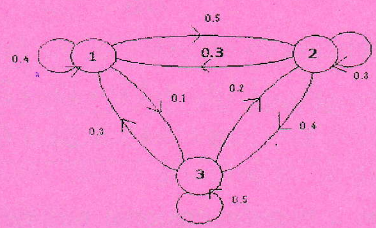
Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

Marks, K-Level,CO

1. If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have high fidelity and selectivity is 0.18. What is the probability that a system with high fidelity will also have selectivity? *2,K3,CO1*
2. If the probability that a target is destroyed on any one shot is 0.5, what is the probability that it would be destroyed on the 6^{th} attempt? *2,K3,CO1*
3. If two random variables X and Y have joint probability density function $f(x,y)=k(2x+y)$ for $0\leq x\leq 2$ and $0\leq y\leq 3$. Evaluate k. *2,K3,CO2*
4. State central limit theorem. *2,K1,CO2*
5. State the four types of stochastic process. *2,K1,CO3*
6. Find the transition probability matrix of the process represented by the state transition diagram *2,K3,CO3*



7. What do the letters in the symbolic representation (a/b/c):(d/e) of a queueing model represent? *2,K1,CO4*
8. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a M/M/1 queueing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour? *2,K3,CO4*
9. Define series queues. *2,K1,CO5*
10. What is bottle neck of a tandem queuing system? *2,K1,CO5*

PART - B ($5 \times 16 = 80$ Marks)

Answer ALL Questions

11. a) i) A random variable X has the following probability function 8,K3,CO1

X = x:	0	1	2	3	4	5	6	7
P(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

a) Find k.

b) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(0 < X < 5)$.

c) Find $P(1.5 < X < 4.5 / X > 2)$.

d) Find the minimum value of 'a' such that $P(X \leq a) > \frac{1}{2}$.

ii) Find the moment generating function of an exponential distribution. Hence find mean and variance. 8,K3,CO1

OR

b) i) A continuous random variable X has p.d.f. $f(x) = kx^2e^{-x}, \ x \geq 0$. Find k, mean and variance. 8,K3,CO1

ii) In a city the daily consumption of electric power in million kilowatts hours is a random variable with Erlang distribution with parameter, $\lambda = \frac{1}{2}$ and $k = 3$. If the power plant of this city has a daily capacity of 12 million kilowatts hours, what is the probability that this power supply will be inadequate on any given day? 8,K3,CO1

12. a) The joint probability mass function of (X,Y) is given by $p(x,y) = k(2x+3y)$, $x=0,1,2$, $y=1,2,3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of $X+Y$. 16,K3,CO2

OR

b) i) Find the correlation co-efficient for the following data: 8,K3,CO2

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

ii) Let (X,Y) be a two-dimensional non-negative continuous random variable having the joint density function $f(x,y) = \begin{cases} 4xy\, e^{-(x^2+y^2)}, & x \geq 0, \ y \geq 0 \\ 0, & elsewhere \end{cases}$. Find the density function of $U = \sqrt{X^2+Y^2}$. 8,K3,CO2

13. a) i) The probability distribution of the process $\{X(t)\}$ is given by 8,K3,CO3

$$P\left[X(t)=n\right]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\ldots\\ \dfrac{at}{1+at}, & n=0\end{cases}$$ Show that it is not stationary.

ii) The one step transition probability matrix of a Markov chain $(X_n\,; n=0,1,2,\ldots)$ having state space $S=(1,2,3)$ is $\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $P^{(0)}=(0.7,0.2,0.1)$. Find 8,K3,CO3

(a) $P(X_2=3/X_0=1)$

(b) $P(X_3=2, X_2=3, X_1=3, X_0=2)$

(c) $P(X_2=3)$.

OR

b) i) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in City A, then the next day, he sells in city B. However, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in the other city. In the long run, how often does he sell in each of the cities? 8,K3,CO3

ii) If the customers arrive in accordance with Poisson process with mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is 8,K3,CO3

a) more than 1 minute,

b) between 1 and 2 minutes,

c) less than 4 minutes.

14. a) i) Arrivals at a telephone booth are considered to be Poisson with an average time 12 minutes between one arrival and the next. The length of telephone call is assumed to be distributed exponentially with mean 4 minutes. 8,K3,CO4

a) Find the average number of persons waiting in the system.

b) What is the probability that a person arriving at the booth has to wait in the queue?

c) Also estimate the fraction of the day when phone will be in use.

ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour. 8,K3,CO4

a) Find the effective arrival rate at the clinic.

b) What is the probability that an arriving patient does not have to wait?

c) What is the expected waiting time until a patient is discharged from clinic?

OR

b) i) A petrol pump has two pumps. The service times follow the exponential distribution with mean 4 minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service, what is the probability that the pumps remain idle? *8,K3,CO4*

ii) A 2-person barber shop has 5 chairs to accommodate waiting customers. Potential customers who arrive when all 5 chairs are full, leave without entering the barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 minutes in the barber's chair. Compute P_0 , P_7 and average number of customers in the queue. *8,K3,CO4*

15. a) Derive Pollaczek – Khintchine formula. *16,K3,CO5*

OR

b) i) An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes determine L_s, L_q, W_s and W_q. *8,K3,CO5*

ii) A repair facility shared by a large number of machines has 2 sequential stations with respective service rates of 2 per hour and 3 per hour. The cumulative failure rate of all the machines is 1 per hour. Assuming that the system behavior may be approximated by the 2-stage series queue. Find *8,K3,CO5*

a) The average repair time including the waiting time.

b) The probability that both the service stations are idle.

c) The bottle neck of the repair facility.