Reg. No.												

Question Paper Code	11709

B.E./B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022

Third Semester

Artificial Intelligence and Data Science

20BSMA302 - PROBABILITY AND STATISTICAL MODELLING

(Regulations 2020)

Duration: 3 Hours Max. Marks: 100

PART - A ($10 \times 2 = 20$ Marks)

Answer ALL Questions

		Marks, K-Level,CO
1.	State Bayes' theorem.	2,K1,CO1
2.	Check whether $f(x)=\frac{1}{1+x^2}$, $-\infty < x < \infty$ is a probability density function or not.	2,K2,CO1
3.	State the Central Limit Theorem.	2,K1,CO2
4.	If the joint p.d.f of (X, Y) is given by $f(x,y)=e^{-(x+y)}$, $x \geq 0, y \geq 0$, find $E(XY)$.	2,K2,CO2
5.	The coefficient of correlation between two variables X and Y is 0.48. The covariance is 36. The variance of X is 16. Find the standard deviation of Y.	2,K2,CO3
6.	Define Type-I and Type-II errors.	2,K1,CO3
7.	Discuss about Sign test.	2,K2,CO4
8.	What is the advantage of Wilcoxon signed rank test in comparison with sign test?	2,K1,CO4
9.	Define Point estimate.	2,K1,CO5
10.	Write down the components/variations of time series.	2,K1,CO5

PART - B ($5 \times 16 = 80$ Marks)

Answer ALL Questions

11. a) (i) Two persons are being considered for the post of the principal of a college. The probabilities that the first and second persons will win are 0.6 and 0.4 respectively. If the first person wins, the probability of introducing the common model exam is 0.8 and the corresponding probability if the second person wins is 0.3. What is the probability that the common model examination will be introduced? *8,K3,CO1*

(ii) A discrete random variable X has the probability function, *8,K3,CO1*

x	1	2	3	4	5	6	7	8
$p(x)$	$2a$	$4a$	$6a$	$8a$	$10a$	$12a$	$14a$	$4a$

(i) Find the value of a (ii) Find $P(X < 3)$ and $P(X \geq 5)$ (iii) Find the distribution function.

OR

b) (i) For a binomial distribution mean and standard deviation are 6 and $\sqrt{2}$ respectively. Find the first two terms of the distribution. 8,K3,CO1

(ii) In a normal distribution 7 percent of the items are below 35 and 11 percent of items are above 63, find mean and standard deviation of the distribution. 8,K3,CO1

12. a) (i) Let $X_1, X_2, \ldots, X_n$ be Poisson variates with parameter $\lambda = 2$. Let $S_n = X_1 + X_2 + \ldots + X_n$, where $n = 75$. Find $P[120 \le S_n \le 160]$ 6,K3,CO2

(ii) Given $f_{XY}(x,y) = \begin{cases} cx(x-y); & 0<x<2, -x<y<x \\ 0 & ;otherwise \end{cases}$. 10,K3,CO2

Find (a) c, (b) $f_X(x)$ (c) $f_{Y/X}(y/x)$ and (d) $f_Y(y)$.

OR

b) (i) Find the covariance between X and Y if the joint probability density of X and Y is $f(x,y) = \begin{cases} 2 & for\ x>0,\ y>0,\ x+y<1 \\ 0 & ;elsewhere \end{cases}$. 8,K3,CO2

(ii) If X and Y have the joint p.m.f. 8,K3,CO2

		X = 0	X = 1	X = 2
Y	0	0.1	0.4	0.1
	1	0.2	0.2	0

Find (i) $P(X + Y > 1)$

(ii) The probability mass function of X.

(iii) $E(XY)$

(iv) $P(Y = 1 / X = 1)$.

13. a) (i) A mathematics test was given to 50 girls and 75 boys. The girls got an average grade of 76 with a S.D. of 6, while boys got an average of 82 with a S.D. of 2. Test whether there is any significant difference between the performance of boys and girls. 8,K3,CO3

(ii) The lines of regression of a bivariate population are: 8,K3,CO3

$8x - 10y + 66 = 0$ and $40x - 18y = 214$. The variance of x is 9. Find

a) The mean values of x and y.

b) Correlation coefficient between x and y

c) Standard deviation of y.

OR

b) A company appoints four salesman A, B, C and D and observes their sales in three seasons: summer, winter and monsoon. The figures (in lakhs of Rs.) are given in the following table. Carry out analysis of variance. 16,K3,CO3

Salesman

		A	B	C	D
Seasons	Summer	45	40	38	37
	Winter	43	41	45	38
	Monsoon	39	39	41	41

14. a) (i) The following are the average weekly losses of workers-hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation: 8,K3,CO4

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

Use 0.05 LOS to test whether the safety program is effective by sign test.

(ii) In an industrial production line items are inspected periodically for defectives. The following is a sequence of defectives items (D) and non- defective items (N) produced by these production line. 8,K3,CO4

DD NNN D NN DD NNNNN DDD NN D NNNN D N D

Test whether the defectives are occurring at random or not at 5% level of significance.

OR

b) (i) The following are the weights in pounds before and after of 16 persons who stayed on a certain reducing diet for four weeks. 8,K3,CO4

Before	147	183.5	232.1	161.6	197.5	206.3	177	215.4
After	137.9	176.2	219	163.8	193.5	201.4	180.6	203.2

Before	147.7	208.1	166.8	131.9	150.3	197.2	159.8	171.7
After	149	195.4	158.5	134.4	149.3	189.1	159.1	173.2

Use Wilcoxon signed rank test to test at 0.05 LOS whether the weight reducing diet is effective.

(ii) The following is the table of observed frequencies representing number of students availing of educational loan from different engineering institutes. Check whether this observation seems to be uniform or not by K-S test at 0.05 LOS. [Use D(0.05, 60) = 0.1756] 8,K3,CO4

Institutions	A	B	C	D	E
Observed frequencies	5	9	11	16	19

15. a) (i) Given a random sample of size n from a population which has the known mean μ and the finite variance σ^2, show that $S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$ is an unbiased estimator of σ^2. 8,K3,CO5

(ii) Find the maximum likelihood estimator for the parameter λ of a Poisson distribution on the basis of a sample size n. 8,K3,CO5

OR

b) (i) If $X_1, X_2, \ldots, X_n$ constitute a random sample of size n from an exponential population, show that $\bar{X}$ is consistent estimator of the parameter θ. 8,K3,CO5

(ii) Explain about the AR(1) process. Find the mean and variance of AR(1) process. 8,K3,CO5