

B.E./B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022

Third Semester

Civil Engineering

(Common to ECE, EEE, EIE, ICE & CCE)

20BSMA301 - LINEAR ALGEBRA, PARTIAL DIFFERENTIAL EQUATIONS
AND TRANSFORMS

(Regulation 2020)

Duration: 3 Hours

Max. Marks: 100

PART-A (10 × 2 = 20 Marks)

Answer ALL Questions

- | | <i>Marks,
K-Level, CO</i> |
|---|-------------------------------|
| 1. Show that x, y and z are vectors in a vector space V such that $x + y = y + z$ then $x = z$. | 2, K2, CO1 |
| 2. Let $V = R^3, S = \{(1, 2, 0), (0, -5, -7)\}$ and $V = (2, -5, 7) \in V$. Verify whether V is a linear combination of S or not. | 2, K2, CO1 |
| 3. Show that T is linear when $T: R^2 \rightarrow R^2$ is defined by $T(a_1, a_2) = (2a_1 + a_2, a_1)$. | 2, K2, CO2 |
| 4. If u and v are vectors in an inner product space. Prove that $\ u + v\ ^2 + \ u - v\ ^2 = 2(\ u\ ^2 + \ v\ ^2)$. | 2, K2, CO2 |
| 5. Find the PDE of all planes having equal intercepts on the x and y axis. | 2, K1, CO3 |
| 6. Form the partial differential equation by eliminating the arbitrary constants ' a ' and ' b ' from $z = ax + by$. | 2, K2, CO3 |
| 7. Find Fourier Sine Transform of e^x , for $x > 0$ | 2, K2, CO4 |
| 8. State Parseval's Identity. | 2, K1, CO4 |
| 9. Define Z- transform. | 2, K1, CO5 |
| 10. Find $Z\{na^n\}$. | 2, K2, CO5 |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) (i) In $R^3(R)$, show that the vectors $(1, 4, -2)$, $(-2, 1, 3)$ and $(-4, 11, 5)$ are linearly dependent and find the relation between them. 8, K3, CO1
(ii) Show that the intersection of two subspaces of a vector space is a subspace. 8, K3, CO1
- OR**
- b) (i) Let W_1 and W_2 be subspaces of a vector space V . Prove that V is the direct sum of W_1 and W_2 if and only if each vector in V can be uniquely written as $x_1 + x_2$ where $x_1 \in W_1$ and $x_2 \in W_2$. 8, K3, CO1

(ii) Prove that $S = \{(1,2,3), (2,3,1)\}$ in $R^3(R)$ is linearly independent over R . 8,K3,CO1

12. a)

(i) Test the matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ for diagonalizability. 8,K2,CO2

(ii) Let V be an inner product space over F . Then for all $u, v \in V$. Show that $|\langle u, v \rangle| \leq \|u\| \|v\|$. 8,K2,CO2

OR

b) Find an orthonormal basis of the inner product space $R^3(R)$ with the standard inner product, given the basis $\{(1, 0, 1), (0, 1, 1), (1, 3, 3)\}$ using Gram-Schmidt process. Also find the Fourier coefficients of the vector $(1, 1, 2)$ relative to the orthonormal basis. 16,K3,CO2

13. a) (i) Form the partial differential equation by eliminating the arbitrary functions f and g in $z = x^2 f(y) + y^2 g(x)$. 8,K3,CO3

(ii) Find the singular solution of $z = px + qy + \sqrt{p^2 + q^2 + 1}$. 8,K3,CO3

OR

b) (i) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. 8,K3,CO3

(ii) Solve $(D^2 + 3DD' + 2D'^2)z = \sin(2x + y) + x$. 8,K3,CO3

14. a) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(y^2 + b^2)}$ using Fourier cosine transform of e^{ax} and e^{bx} . 16,K3,CO4

OR

b) Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x| & \text{for } |x| > 1 \\ 0 & \text{for } |x| < 1 \end{cases}$. Hence 16,K3,CO4

deduce that $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$.

15. a) (i) Find $Z^{-1} \left[\frac{8z^2}{(2z-1)(4z-1)} \right]$ using convolution. 8,K3,CO5

(ii) Find $Z[n(n-1)(n-2)]$. 8,K3,CO5

OR

b) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transforms. 16,K3,CO5