Reg. No.												

Question Paper Code	11713

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022

Third Semester

Mechanical Engineering

(Common to Mechanical and Automation Engineering)

20BSMA303 - PARTIAL DIFFERENTIAL EQUATIONS AND PROBABILITY THEORY

(Regulations 2020)

(Usage of Statistical Table Permitted)

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

	Question	Marks, K-Level, CO
1.	Form the PDE by eliminating the arbitrary constants a, b from $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$.	2,K3,CO1
2.	Solve $p+q=pq$.	2,K3,CO1
3.	Classify the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2$.	2,K2,CO2
4.	A rod 20 cm long has its ends A and B kept at $20°C$ and $60°C$ respectively, until steady state conditions prevail. Find the steady state temperature in the rod.	2,K3,CO2
5.	Find the sine transform of $\frac{1}{x}$.	2,K3,CO3
6.	Find the Fourier cosine transform of e^{-x}.	2,K1,CO3
7.	State memory less property for an exponential distribution.	2,K1,CO4
8.	The cdf of the continuous random variable X is given by $F(x)=\begin{cases}0 & x<0\\ 1-e^{-x/5} & x\geq 0\end{cases}$. Find the pdf of X.	2,K2,CO4
9.	Let X and Y be two discrete random variable with joint pmf $P[X=x,Y=y]=\begin{cases}\frac{x+2y}{18}, & x=1,2;\ y=1,2\\ 0 & ,otherwise\end{cases}$. Find the marginal pmf of X.	2,K3,CO5
10.	If $y=2x-3$ and $y=5x+7$ are the two regression lines, find the mean values of x and y.	2,K3,CO5

PART - B (5 × 16 = 80 Marks)
Answer ALL Questions

11. a) (i) Solve $z = px + qy + p^2q^2$. 8,K3,CO1

(ii) Solve $\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right)z = e^{2x-y} + 3$. 8,K3,CO1

OR

b) (i) Solve $\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x + 2y) + e^{2x+y}$. 8,K3,CO1

(ii) Solve $(mz - ny)p + (nx - lz)q = ly - mx$. 8,K3,CO1

12. a) A tightly stretched string with fixed end points $x = 0$ *and* $x = l$ is initially in a position given by $y(x, 0) = y_0 sin^3\left(\frac{\pi x}{l}\right)$. If it is released from the rest from this position, find the displacement y at any time and at any distance from the end $x = 0$. 16,K3,CO2

OR

b) A rectangular plate with insulated surface is 20cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. the temperature at shot edge $x = 0$ is given by $u(y) = \begin{cases} 10y, & 0 \le y \le 10 \\ 10(20 - y), & 10 \le y \le 20 \end{cases}$ and all other three edges are kept at $0°C$. Find the steady state temperature at any point in the plate. 16,K3,CO2

13. a) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin s \cos sx}{s} ds$ *and* $\int_0^\infty \frac{\sin s}{s} ds$. 16,K3,CO3

OR

b) (i) Evaluate $\int_0^\infty \frac{x^2}{(x^2+16)(x^2+9)} dx$ using transform techniques. 8,K3,CO3

(ii) Show that the Fourier transform of $e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$. 8,K3,CO3

14. a) (i) The contents of urns I, II and III are as follows: 1 white, 2 black and 3 red balls, 2 white, 1 black and 1 red ball, and 4 white, 5 black and 3 red balls respectively. One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urn I. 8,K3,CO4

(ii) In a company the monthly break down of a machine is a random variable with Poisson distribution, with an average 1.8. Find the probability that the machine will function for a month (1) Without break down, (2) With exactly one break down, (3) With at least one break down. 8,K3,CO4

OR

b) (i) A random variable X has the following probability function : 8,K3,CO4

Value of x	0	1	2	3	4	5	6	7
$P(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(1) Find k.

(2) Evaluate $P(X < 6), P(X \geq 6), P(0 < X < 5)$.

(3) Determine the distribution function of X .

(ii) The number of accidents in a year attributed to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, Find approximately the number of drivers with 8,K3,CO4

(1) no accident in a year

(2) more than 3 accidents in a year.

15. a) The joint probability mass function of (X,Y) is given by $p(x,y) = k(2x+3y)$, $x=0,1,2, y=1,2,3$. Find all the marginal and conditional probability distributions. 16,K3,CO5

OR

b) (i) If the joint pdf of (X,Y) is given by $f_{XY}(x,y) = e^{-(x+y)}$; $x \geq 0$, $y \geq 0$, find the pdf of $U = X + Y$. 8,K3,CO5

(ii) If X is a normal variate with $\mu = 30$ and $\sigma = 5$, find 8,K3,CO5

(1) $P(26 \leq X \leq 40)$ (2) $P(X \geq 45)$ (3) $P(|X - 30| \geq 5)$.