

Reg. No.

Question Paper Code

11733

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022

Second Semester

Artificial Intelligence and Data Science

(Common to Computer Science and Engineering, Information Technology & M.Tech. - Computer Science and Engineering)

20BSMA204 - DISCRETE STRUCTURES

(Regulations 2020)

Duration: 3 Hours

Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

- | | <i>Marks,
K-Level, CO</i> |
|---|-------------------------------|
| 1. Let $A = \{1, 2, 3, 4\}$ then the relation $R = \{(1, 1), (2, 4), (2, 2), (3, 3), (4, 1), (4, 4)\}$. Check whether R is symmetric or not. | 2,K2,CO1 |
| 2. If $f = \{(1, 3), (2, 5), (3, 1), (4, 2), (5, 4)\}$ then find f^{-1} . | 2,K3,CO1 |
| 3. How many bit strings of length 10 contain (i) at most four 1's
(ii) at least four 1's. | 2,K3,CO2 |
| 4. In a group of 100 people, several will have birth days in the same month, at least how many of them must have birth days in the same month? | 2,K3,CO2 |
| 5. What are the contrapositive, the converse and the inverse of the conditional statement "the home team wins whenever it is raining". | 2,K2,CO3 |
| 6. Write the dual of the statement $\neg(p \vee q) \vee [(\neg p) \wedge q] \vee p$. | 2,K2,CO3 |
| 7. Define order of a group. | 2,K1,CO4 |
| 8. In any Boolean algebra, prove that $(a+b).(a'+c) = ac + a'b + bc$. | 2,K2,CO4 |
| 9. Define a complete graph and give an example. | 2,K1,CO5 |
| 10. State and prove hand shaking theorem. | 2,K1,CO5 |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) (i) If R is the relation on the set of integers such that $(a, b) \in R$ iff $3a + 4b = 7n$ for some integer n , show that R is an equivalence relation. 8,K3,CO1
- (ii) If R and S are given by $R = \{(1, 2), (2, 2), (3, 4)\}$,
 $S = \{(1, 3), (2, 5), (3, 1), (4, 2)\}$ then determine $R \circ S$, $S \circ S$, $(R \circ R) \circ R$ &
 $R \circ (S \circ R)$. 8,K3,CO1

OR

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create

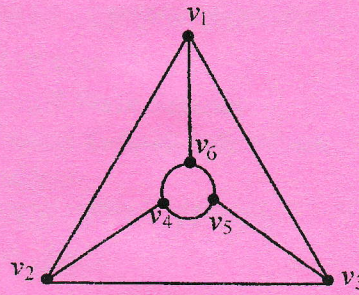
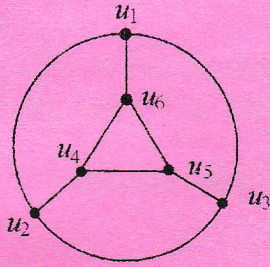
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- b) (i) If $f, g, h: R \rightarrow R$ are defined by $f(x) = x^3 - 4x$, $g(x) = \frac{1}{x^2 + 1}$ and $h(x) = x^4$, Determine $f \circ g$, $g \circ h$, $f \circ h$. 8, K3, CO1
- (ii) Show that the inclusion relation ' \subseteq ' is a partial ordering on the power set of a set S. 8, K3, CO1
12. a) (i) Prove the following by using Mathematical induction $(3^n + 7^n - 2)$ is divisible by 8, for $n \geq 1$. 8, K3, CO2
- (ii) How many integers between 1 and 300 (both inclusive) are divisible by
 1) at least one of 3, 5, 7
 2) 3 and 5 but not 7? 8, K3, CO2
- OR**
- b) (i) Prove the following by using Mathematical induction $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \geq 2$. 8, K3, CO2
- (ii) How many permutations of the letters A, B, C, D, E, F, G contain
 1) string BCD, 2) the strings BA and GF, 3) the strings ABC and CDE,
 4) the strings CBS and BED? 8, K3, CO2
13. a) (i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$. 8, K3, CO3
- (ii) Show that the hypothesis "x works hard", "if x works hard then he is a dull boy" and "if x is a dull boy then he will not get a job" imply the conclusion "x will not get a job". 8, K3, CO3
- OR**
- b) (i) Using the rule CP or otherwise show the following implications $(\exists x)P(x) \rightarrow (x)Q(x) \Rightarrow (x)(P(x) \rightarrow Q(x))$ 8, K3, CO3
- (ii) Show that the premises "Everyone in the computer science branch has studied discrete mathematics" and "Ram is in computer science branch" imply that "Ram has studied discrete mathematics". 8, K3, CO3
14. a) State and prove Lagrange's theorem. 16, K3, CO4
- OR**
- b) (i) The necessary and sufficient condition for a non-empty subset H of a group $(G, *)$ to be a subgroup is $a, b \in H \Rightarrow a * b^{-1} \in H$. 8, K3, CO4
- (ii) If $*$ defined on R such that $a * b = a + b - ab$, $a, b \in R$. Show that $(R, *)$ is an abelian group. 8, K3, CO4

15. a) A non-empty connected graph G is Eulerian iff its vertices are of even degree. 16, K3, CO5

OR

- b) (i) Explain isomorphism. Test whether the graphs G_1 and G_2 are isomorphic or not. 8, K3, CO5



- (ii) The maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. 8, K3, CO5