B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022 (MARCH 2023)

First Semester

## Computer Science and Business Systems 20BSMA102 - DISCRETE MATHEMATICS

(Regulations 2020)
Duration: 3 Hours
Max. Marks: 100

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\text { PART - A }(10 \times 2=20 \text { Marks })
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Answer ALL Questions

Marks,<br>K-Level, CO

1. Express using logical connectives, "you access the internet from campus $2, \mathrm{~K}, \mathrm{CO}$ only if you are a computer science major or you are not a freshman".
2. Show that $-(p \wedge q) \Rightarrow(-p \vee-q)$.
$2, \mathrm{~K}, \mathrm{CO} 1$
3. How many 16 -bit strings are there containing exactly 5 zeros?
4. Find the recurrence relation for $y_{n}=A \cdot 2^{n}+B \cdot 3^{n}$.
5. What is the Boolean expression for NAND gate?
6. Show that in a Boolean Algebra, $a \leq b \Rightarrow a+(b \cdot c)=b \cdot(a+c)$. 2,K2,CO3
7. Draw a graph with 5 vertices $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$ such that $\operatorname{deg}(\mathrm{A})=3, \mathrm{~B}$ is an odd $2, \mathrm{K2}, \mathrm{CO} 4$ vertex, $\operatorname{deg}(C)=2, D$ and $E$ are adjacent.
8. Define chromatic number of a graph G with example.

2,K1,CO4
9. Define a group with proper example.

2,KI,CO5
10. Define a Ring with Example.

2,KI,COS

## PART - B ( $5 \times 16=80$ Marks $)$ <br> Answer ALL Questions

11. a) (i) Construct the truth table for the following

8, K2, COI
$\neg(P \vee(Q \wedge R)) \leftrightarrow((P \vee Q) \wedge(P \vee R))$.
(ii) Show that $((p \vee q) \wedge(\neg p \vee r)) \rightarrow(q \vee r)$ is a tautology. OR
b) (i) Find the CNF \& DNF of $P \rightarrow((P \rightarrow Q) \wedge(\neg(\neg Q \vee \neg P)))$ using laws of logic.
(ii) Find the PCNF \& PDNF of $P \rightarrow(Q \wedge P) \wedge(\neg P \rightarrow(\neg Q \wedge \neg R))$ using laws of logic.
12. a) Find the recurrence formula for the Fibonacci sequence of numbers 16,K3, CO2 and obtain its solution.

## OR

b) State the Pigeon hole principle and also prove that there exists a positive integer $n$ such that $m$ divides $2^{n}-1$ where $m$ being a positive odd integer.
13. a) (i) Prove that in a Boolean Algebra,
$8, \mathrm{~K} 3, \mathrm{CO} 3$ $\left(a+b^{\prime}\right) \cdot\left(b+c^{\prime}\right) \cdot\left(c+a^{\prime}\right)=\left(a^{\prime}+b\right) \cdot\left(b^{\prime}+c\right) \cdot\left(c^{\prime}+a\right)$.
(ii) Prove that in a Boolean Algebra, $a+a^{\prime} b c^{\prime}+(b+c)^{\prime}=a+c^{\prime}$.

8, K3, CO3 OR
b) Minimize the following Boolean function using K-Map.
$16, K 3, \mathrm{CO} 3$
$F=A^{\prime} B^{\prime} C^{\prime}+A^{\prime} B^{\prime} C+A B^{\prime} C+A B C^{\prime}$.
14. a) (i) Check whether the following graphs are isomorphic or not.

8, K3,CO4

(ii) Consider the following digraph. Use its adjancy matrix to find how many paths of length 3 from V1 to V2.


## OR

b) (i) Prove that the maximum number of edges in a simple 8,K3,CO4 disconnected graph $G$ with $n$ vertices and $k$ components is $\frac{(n-k)(n-k+1)}{2}$.
(ii) Prove that a graph is a tree if and only if there is a unique simple

8,K3,CO4 path between every pair of vertices.
15. a) State and Prove Fundamental theorem of Group homomorphism. $16, K 3, \operatorname{CO5}$

## OR

b) State and Prove Lagrange's Theorem.
$16, K 3, \operatorname{CO} 5$

