| Reg. No. | | | | | | | | | | | | |
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| Question Paper Code | 12010 |
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17 JUL 2023

**B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL/MAY 2023**

Fourth Semester

**Computer Science and Engineering**

(Common to Information Technology & M.Tech- Computer Science and Engineering)

**20BSMA402 - PROBABILITY AND QUEUEING THEORY**

(Regulations 2020)

(Use of Statistical Table is permitted)

Duration: 3 Hours | Max. Marks: 100

**PART-A (10 × 2 = 20 Marks)**

Answer ALL Questions

***Marks, K-Level, CO***

1. If a random variable $X$ has the moment generating function $M(t)=\frac{3}{3-t}$, obtain the standard deviation of $X$. *2,K2,CO1*
2. The probability that a target is destroyed in any one shot is 0.5. What is the probability that it would be destroyed on $7^{th}$ attempt? *2,K2,CO1*
3. Find the value of 'k', if the joint density function of $(X,Y)$ is given by *2,K2,CO2*
$$f(x,y)=\begin{cases} k(1-x)(1-y), & 0<x<4, 1<y<5 \\ 0 & , \text{otherwise} \end{cases}$$
4. The regression equation of $X$ on $Y$ and $Y$ on $X$ are $5x-y=22$ and $64x-45y=24$ respectively. Find the means of $X$ and $Y$. *2,K2,CO2*
5. Prove that the Poisson process is a Markov process. *2,K1,CO3*
6. Define Markov chain. *2,K1,CO3*
7. Explain Kendall's notations. *2,K1,CO4*
8. What is the effective arrival rate for $(M|M|1):(4|FIFO)$ queuing model when $\lambda=2$ and $\mu=5$? *2,K2,CO4*
9. A M/D/1 queue has an arrival rate 10 customers per second and a service rate of 20 customers per second. Calculate the mean number of customer in the queue. *2,K2,CO5*
10. Define series queues. *2,K1,CO5*

**PART - B (5 × 16 = 80 Marks)**

Answer ALL Questions

11. a) (i) A factory has two machines I and II. Machine I and II produce 30% and 70% of items respectively. Further 3% of items produced by machine I are defective and 4% of items produced by machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by machine II. *8,K3,CO1*

(ii) A random variable $X$ has the following probability distribution 8,K3,CO1

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $P(x)$ | $a$ | $3a$ | $5a$ | $7a$ | $9a$ | $11a$ | $13a$ | $15a$ | $17a$ |

Find the value of '$a$', $P(X < 3)$ , $P(0 < X < 5)$.

**OR**

b) Find the moment generating function of Binomial distribution and hence find its mean and variance. 16,K3,CO1

12. a) The joint probability mass function of $(X, Y)$ is given by $P(x, y) = K(2x + 3y),\ x = 0, 1, 2;\ y = 1, 2, 3$. Find all the marginal and conditional probability distributions. 16,K3,CO2

**OR**

b) (i) Find the coefficient of correlation between industrial production and export using the following data. 8,K3,CO2

| **Production (x)** | 55 | 56 | 58 | 59 | 60 | 60 | 62 |
|---|---|---|---|---|---|---|---|
| **Export (y)** | 35 | 38 | 37 | 39 | 44 | 43 | 44 |

(ii) If $X_1, X_2, X_3, ..., X_n$ are Poisson variates with parameter $\lambda = 2$, use the Central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + X_3 + .. + X_n$ and $n = 75$. 8,K3,CO2

13. a) (i) Prove that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sense stationary if A & $\omega_0$ are constants and $\theta$ is a uniformly distributed random variable in $(0, 2\pi)$. 8,K3,CO3

(ii) If the customers arrive at a counter in accordance with a Poisson process with mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is 8,K3,CO3

(a) more than 1 minute.

(b) between 1 and 2 minutes.

(c) 4 minutes or less.

**OR**

b) A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B or in C, then the next day he is thrice as likely to sell in city A as in other city. In the long run, how often does he sell in each of the cities? 16,K3,CO3

14. a) Customers arrive at a one – man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. If an hour is used as a unit time, then 16,K3,CO4

1) What is the probability that a customer need not wait for a haircut?

2) What is the expected number of customers in the barber shop and in the queue?

3) How much time can a customer expect to spend in the barber shop?

4) Find the average time that the customer spends in the queue.

5) Estimate the fraction of the day that the server will be idle?

**OR**

b) Suppose there are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive to be typed at the rate of 15 letters per hour, *16,K3,CO4*

1) What fraction of the time all the typists will be busy?
2) What is the average number of letters waiting to be typed?
3) What is the probability that one letter in the system?
4) What is the average time a letter has to spend in the system?

15. a) Derive Pollaczek - Khinchine formula of M/G/1 queuing model. *16,K3,CO5*

**OR**

b) Consider an open Jackson network with parameter values shown below: *16,K3,CO5*

$P_{ij}$

| Facility $j$ | $c_j$ | $\mu_j$ | $r_j$ | $i=1$ | $i=2$ | $i=3$ |
|---|---|---|---|---|---|---|
| $j=1$ | 1 | 10 | 1 | 0 | 0.1 | 0.4 |
| $j=2$ | 2 | 10 | 4 | 0.6 | 0 | 0.4 |
| $j=3$ | 1 | 10 | 3 | 0.3 | 0.3 | 0 |

1) Find the steady state distribution of the number of customers at facility 1, facility 2 and facility 3.
2) Find the expected number of customers in the system.
3) Find the expected total waiting time in the system.