17 JUL 2023

Reg. No. | | | | | | | | | | | | |

| Question Paper Code | 12012 |
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**B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL/MAY 2023**

Fourth Semester

**Artificial Intelligence and Data Science**

**20BSMA404 - LINEAR ALGEBRA AND ITS APPLICATIONS**

(Regulations 2020)

Duration: 3 Hours | Max. Marks: 100

## PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

| | | *Marks, K-Level, CO* |
|---|---|---|
| 1. | Define linearly independent set vectors. | 2,K1,CO1 |
| 2. | Write the vector (2,-5,3) as a linear combination of the vectors (1,-3, 2 ), (2,-4,-1) &(1,-5,7). | 2,K2,CO1 |
| 3. | Prove that in any vector space a.0 = 0 for each a ∈ F. | 2,K1,CO2 |
| 4. | Define linearly independent vectors. | 2,K1,CO2 |
| 5. | Examine the map T: R→R defined by T(x) = x+3 , $x \in R$ is a linear transformation. | 2,K1,CO3 |
| 6. | Define kernel of a linear transformation. | 2,K1,CO3 |
| 7. | Prove that $< x, cy > = \bar{c} < x, y >$. | 2,K1,CO4 |
| 8. | Let V be an inner product space, and suppose that x and y are orthogonal vectors in V. prove that $\|x+y\|^2 = \|x\|^2 + \|y\|^2$. | 2,K2,CO4 |
| 9. | Define Right singular Vector. | 2,K2,CO5 |
| 10. | Why we use SVD and PCA in Data Science. | 2,K2,CO5 |

## PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) (i) Find the value of a and b if the matrix $A = \begin{pmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 4 \\ 7 & -1 & a & b \end{pmatrix}$ is of rank 2. — *8,K3,CO1*

(ii) Solve by LU Decomposition method — *8,K3,CO1*

$$3x_1 - 6x_2 - 3x_3 = -3$$
$$2x_1 + 6x_3 = -22$$
$$-4x_1 + 7x_2 + 4x_3 = 3$$

**OR**

b) (i) Solve by Cramer's rule *8,K3,CO1*

$$10x + y - z = 12$$
$$2x + 10y - z = 13$$
$$2x + 2y - 10z = 14$$

(ii) Solve the system by Gauss Elimination Method *8,K3,CO1*

$$5x_1 + 3x_2 + 7x_3 = 4$$
$$3x_1 + 26x_2 + 2x_3 = 9$$
$$7x_1 + 2x_2 + 10x_3 = 5$$

12. a) Show that $R^n = \{(x_1, x_2, x_3, \dots, x_n) : x_i \in R\}$ is a vector space over F with respect to addition and scalar multiplication defined component wise. *16,K3,CO2*

**OR**

b) (i) Prove that the intersection of two subspaces of a vector space V is again a subspace of V. *8,K3,CO2*

(ii) Determine whether the set *8,K3,CO2*
$W = \{(1,0,-1), (2,5,1), (0,-4,3)\} \subseteq R^3$ is a basis for $R^3$.

13. a) (i) Prove that there exists a linear transformation T: $R^2 \rightarrow R^3$ such that T (1,1) = (1,0,2) and T (2,3) = (1, -1,4). What is T (8,11)? *8,K3,CO3*

(ii) Let V and W be vector spaces and T: V→ $W$ be linear. Then prove that T is one-one if and only if N(T) = {0}. *8,K3,CO3*

**OR**

b) State and prove Dimension Theorem. *16,K3,CO3*

14. a) (i) State and prove Cauchy-Schwarz inequality. *8,K3,CO4*

(ii) Compute the least square solution of the equations *8,K3,CO4*

x+5y=3
2x-2y =2
-x +y =5 .

Also find the least square error.

**OR**

b) (i) State and prove Gram-Scdmidth orthogonalization process. *8,K3,CO4*

(ii) Determine the QR-Decomposition of the matrix A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ *8,K3,CO4*

15. a) (i) Determine the matrix U, Σ,V such that A=UΣV$^T$, where *8,K3,CO5*

$$A=\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}.$$

(ii) Discuss the applications of Linear Algebra in Data Science. *8,K3,CO5*

**OR**

b) (i) Compute $A^TA$ and $AA^T$, their eigen values and unit eigen vector $v$ and $u$ for the rectangular matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and compute SVD of the matrix $A=U\Sigma V^T$. *8,K3,CO5*

(ii) Suppose $A_0$ has these two measurements of 5 samples. $A_0= \begin{bmatrix} 1 & 0 & -1 & 1 & 2 & 3 \\ 0 & 1 & 0 & 3 & 2 & 1 \end{bmatrix}$ Compute the centred matrix A, sample covariance S, eigen values $\lambda_1, \lambda_2$ . what is the line through the origin is closest to the 5 sample in the column of A. *8,K3,CO5*