

Reg. No.

Question Paper Code

12139

14 AUG 2023

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2023

First Semester

Civil Engineering

(Common to All Branches except Computer Science and Business Systems)

20BSMA101 – ENGINEERING MATHEMATICS - I

(Regulations 2020)

Duration: 3 Hours

Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

- | | <i>Marks,
K-Level, CO</i> |
|---|-------------------------------|
| 1. The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ is 16. Find the third eigen value of A. | 2,K2,CO1 |
| 2. Write down the matrix of the quadratic form $2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1x_2 - 6x_1x_3 + 6x_2x_3$. | 2,K2,CO1 |
| 3. State Sandwich Theorem. | 2,K1,CO2 |
| 4. Find $\lim_{x \rightarrow 0} \frac{ x }{x}$ if it exists. | 2,K2,CO2 |
| 5. Evaluate $\int (x+3)(x-2) dx$. | 2,K2,CO3 |
| 6. Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x dx$. | 2,K2,CO3 |
| 7. Evaluate $\int_0^1 \int_1^2 x(x+y) dy dx$. | 2,K2,CO4 |
| 8. Sketch roughly the region of integration for $\int_0^1 \int_0^x f(x,y) dy dx$. | 2,K2,CO4 |
| 9. Find the Taylor's series representation of $f(x) = e^x$ about $x = 0$. | 2,K2,CO5 |
| 10. Find the Fourier constant b_n for $x \sin x$ in $(-\pi, \pi)$. | 2,K2,CO5 |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ to the canonical form through an orthogonal transformation. Hence find rank, index, signature and nature of the quadratic form. 16,K3,CO1
- OR**
- b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ 16,K3,CO1
and hence find A^{-1} and A^4 .

12. a) (i) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. 8,K3,CO2

(ii) Examine $f(x, y) = x^3 + y^3 - 12x - 3y + 20$ for its extreme values. 8,K3,CO2

OR

- b) (i) A rectangular box, open at the top, is to have a volume of 32cc. Find dimensions of box which least amount of material for its construction. 8,K3,CO2

(ii) Expand $x^2y + 3y - 2$ in powers of $x - 1$ and $y + 2$ using Taylor's theorem. 8,K3,CO2

13. a) Prove that the reduction formula for $I_n = \int \sin^n x \, dx$ is 16,K3,CO3

$$I_n = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}. \text{ Hence find } \int_0^{\frac{\pi}{2}} \sin^n x \, dx.$$

OR

- b) (i) Evaluate $\int \frac{dx}{x^2\sqrt{x^2-1}}$. 8,K3,CO3

(ii) Evaluate $\int \frac{x}{(1+x)(1+x^2)} dx$. 8,K3,CO3

14. a) (i) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. 8,K3,CO4

(ii) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ changing the order of integration 8,K3,CO4

OR

- b) (i) Find the volume of sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals 8,K3,CO4

(ii) Evaluate $\iint r^3 dr d\theta$, over the area bounded between the circles $r = 2 \sin \theta$ and $r = 4 \sin \theta$. 8,K3,CO4

15. a) Find the cosine series for $f(x) = x$ in $(0, \pi)$ and deduce 16,K3,CO5

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots = \frac{\pi^4}{96}.$$

OR

- b) Find the Fourier series for $f(x) = x^2$ in $-\pi \leq x \leq \pi$ and deduce that 16,K3,CO5

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}.$$