23/01/2023

Reg. No.												

Question Paper Code	11665

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022

Fourth Semester

Electronics and Communication Engineering

(Common to Computer and Communication Engineering)

20BSMA401 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

(Regulations 2020)

(Usage of Normal Table Permitted)

Duration: 3 Hours   Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

Marks, K-Level,CO

1. State any two properties of moments. *2,K1,CO1*
2. The time in hours to repair a machine is exponentially distributed with parameter $\lambda = 1/3$. What is the probability that the repair time exceeds 3 hours? *2,K3,CO1*
3. If X and Y are independent then show that Cov(X,Y) = 0. *2,K1,CO2*
4. State central limit theorem of Lindberg-Levy's form. *2,K1,CO2*
5. Compute the mean of the random process X(t) whose auto correlation function is given by $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$. *2,K3,CO3*
6. State any two properties of power spectral density. *2,K1,CO3*
7. Suppose that customers arrive at a bank according to a Poisson Process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes exactly 4 customers arrive. *2,K3,CO4*
8. Define Markov chain and one-step transition probability. *2,K1,CO4*
9. What is unit impulse response of a system? Why it is called so? *2,K2,CO5*
10. If the input and output of the system $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)\,du$ are wide sense stationary processes, how are their power spectral density function's related? *2,K2,CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) A random variable X has the following probability function: *16,K3,CO1*

X:	0	1	2	3	4	5	6	7
Y:	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K

Find (i) K (ii) $P(X < 6)$ (iii) $P(X \geq 6)$ (iv) $P(0 < X < 5)$ (v) distribution function of X (vi) $Var(3X - 4)$

OR

b) Buses arrive at a specified stop at 15 min. intervals starting at 7 a.m , that is , they arrive at 7,7:15,7:45 and so on . if the passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 a.m. find the probability that he waits (1) less than 5 min for a bus and (2) at least 12 min for a bus. *16,K3,CO1*

12. a) If the joint p.d.f of (X, Y) is given by $f(x, y) = x + y \; for \; 0 < x, y < 1$, find the correlation coefficient between X and Y. *16,K3,CO2*

OR

b) If $X_1, X_2, X_3, \ldots . X_n$ are Poisson variates with parameter $\lambda = 2$, use central limit theorem to estimate $P(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + \cdots + X_n$ and $n = 75$. *16,K3,CO2*

13. a) Show that the random process $X(t) = \cos(t + \phi)$ where ϕ is uniformly distributed in $(0, 2\pi)$ with probability density function $f(\phi) = \frac{1}{2\pi}, 0 < \phi < 2\pi$ is (i) First order stationary (ii) Stationary in wide sense (iii) Ergodic. *16,K3,CO3*

OR

b) A random process {X(t)} is given by X(t) = A cospt + B sinpt, where A and B are independent random variables such that E(A) = E(B) = 0 and $E(A^2) = E(B^2) = \sigma^2$.Find the power spectral density of the process. *16,K3,CO3*

14. a) Suppose that customers arrive at a bank according to a Poisson Process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes *16,K3,CO4*
   (i) exactly 4 customers arrive.
   (ii) more than 2 customers arrive.
   (iii) atmost 2 customers arrive.

OR

b) A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov Chain $\{X_n\}$. Find also $P\{X_2=6\}$. *16,K3,CO4*

15. a) A random process X(t) is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t \geq 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, determine the cross correlation function between the input process X (t) and the output process Y (t). *16,K3,CO5*

OR

b) If the input X(t) and the output Y(t) are connected by the differential equation $T\frac{dy(t)}{dt} + y(t) = x(t)$, prove that they can be related by means of a convolution type integral. Assume that x (t) and y (t) is zero for $t \leq 0$. *16,K3,CO5*

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create **11665**