

Reg. No.

Question Paper Code

11731

B.E/B.Tech - DEGREE EXAMINATIONS, NOV/DEC 2022

Second Semester

Civil Engineering

(Common to all branches)

20BSMA201 - ENGINEERING MATHEMATICS - II

(Regulations 2020)

Duration: 3 Hours

Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

- |   | <i>Marks,<br/>K-Level, CO</i> |
|---|-------------------------------|
| 1. Find the directional derivative of the function $\phi = x^3yz$ at the point (1, 4, 1).   | 2,K2,CO1                      |
| 2. Prove that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.  | 2,K1,CO1                      |
| 3. Reduce the equation $(x^2D^2 - xD + 4)y = x^2 \sin(\log x)$ into an ordinary differential equation with constant coefficients. | 2,K1,CO2                      |
| 4. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$ .   | 2,K3,CO2                      |
| 5. Show that an analytic function with constant imaginary part is constant.   | 2,K1,CO3                      |
| 6. Find the fixed points of the transformation $w = \frac{6z-9}{z}$ .   | 2,K2,CO3                      |
| 7. Discuss the nature of singularities of $\frac{\sin z - z}{z^3}$ .  | 2,K1CO4                       |
| 8. Find the Taylor's series for $f(z) = \sin z$ about $z = \frac{\pi}{4}$ .   | 2,K2,CO4                      |
| 9. If $L[f(t)] = F(s)$ , then prove that $L[f(e^{at}f(t))] = F(s-a)$ .  | 2,K1,CO5                      |
| 10. Find $L(\cos^2 2t)$ .   | 2,K2,CO5                      |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) Verify Gauss Divergence theorem for  $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$  over the cube bounded by  $x=0, x=1, y=0, y=1, z=0, z=1$ . 16,K3,CO1
- OR**
- b) (i) Find  $a$  and  $b$  so that the surfaces  $ax^3 - by^2z - (a+3)x^2 = 0$  and  $4x^2y - z^3 - 11 = 0$  cut orthogonally at the point (2, -1, -3). 8,K2,CO1

K1 - Remember; K2 - Understand; K3 - Apply; K4 - Analyze; K5 - Evaluate; K6 - Create

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(ii) Verify Green's theorem in a plane for  $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$  8,K2,CO1

where  $C$  is the boundary of the region defined by the lines  $x = 0$ ,  
 $y = 0$  and  $x + y = 1$ .

12. a) (i) Solve  $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin[\log(1+x)]$ . 8,K3,CO2

(ii) Solve  $(x^2 D^2 - xD + 1)y = x \log x$ . 8,K3,CO2

OR

b) (i) Solve  $(D^2 + a^2)y = \tan ax$ , by using method of variation of parameters. 8,K3,CO2

(ii) Solve  $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 4y = x^2 + \frac{1}{x^2}$ . 8,K3,CO2

13. a) (i) If  $f(z)$  is an analytic function of  $z$ , prove that  $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$ . 8,K2,CO3

(ii) Determine the analytic function  $w = u + iv$  if  $u = e^{2x}(x \cos 2y - y \sin 2y)$ . 8,K3,CO3

OR

b) (i) Find the image in the  $w$ -plane of the infinite strip  $\frac{1}{4} \leq y \leq \frac{1}{2}$  under 8,K2,CO3

the transformation  $w = \frac{1}{z}$

(ii) Find the bilinear map which maps the points  $z = 0, -1, i$  into  $w = i, 0, \infty$ . Also find the image of the unit circle of the  $z$ -plane. 8,K2,CO3

14. a) (i) Using Cauchy's integral formula, evaluate  $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$  where 8,K3,CO4

$C$  is  $|z| = 3$ .

(ii) Find the Laurent's series function  $f(z) = \frac{7z-2}{z(z-2)(z+1)}$  in the interval  $1 < |z+1| < 3$ . 8,K2,CO4

OR

b) (i) Evaluate by using Cauchy's residue theorem  $\int_C \frac{z+1}{(z-3)(z-1)} dz$  8,K3,CO4

where  $C$  is the circle  $|z| = 2$ .

(ii) Evaluate  $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$  using contour integration.

8,K3,CO4

15. a)

(i) Find the Laplace transform of  $f(t) = \begin{cases} t & \text{for } 0 < t < a \\ 2a - t, & \text{for } a < t < 2a \end{cases}$  and  
 $f(t + 2a) = f(t)$ .

8,K2,CO5

(ii) Find  $L[t^2 e^{-3t} \sin 2t]$ .

8,K2,CO5

**OR**

b)

(i) Using Laplace transform, solve  $\frac{d^2 y}{dt^2} + 4y = \sin 2t$  given  $y(0) = 3$ ,  
 $y'(0) = 4$ .

8,K3,CO5

(ii) Find  $L^{-1} \left[ \frac{s^2}{(s^2 + 4)^2} \right]$ , using convolution theorem.

8,K2,CO5