Reg. No. | | | | | | | | | | | | |

27 FEB 2023

| Question Paper Code | 11732 |
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**B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV/DEC 2022**

Second Semester

**Computer Science and Business Systems**

**20BSMA202 - LINEAR ALGEBRA**

(Regulations 2020)

Duration: 3 Hours | Max. Marks: 100

**PART - A (10 × 2 = 20 Marks)**

Answer ALL Questions

*Marks, K-Level,CO*

1. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, then find the value of $x$. *2,K3,CO1*
2. Without expanding the determinant, evaluate $\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$. *2,K3,CO1*
3. Test whether the vectors $v_1 = (1, 3, 5)$, $v_2 = (2,5,9)$ and $v_3 = (-3,9,3)$ are linearly dependent or not? *2,K3,CO2*
4. Define rank of a matrix. *2,K1,CO2*
5. Let $V = \{(a_1, a_2): a_1, a_2 \in R\}$ be a vector space over R. Test whether the subset $W = \{(a_1, a_2) : 2a_1 + 3a_2 = 0; a_1, a_2 \in R\}$ is subspace or not. *2,K2,CO3*
6. Show that the set S={(1,2),(3,4)} forms a basis of $R^2$. *2,K2,CO3*
7. Test the map $T: R \rightarrow R$ defined by $T(x) = x + 3 \; \forall \; x \in R$ is a linear transformation. *2,K2,CO4*
8. Let $T: R^2 \rightarrow R^2$ be defined by $T(x, y) = (4x - 2y, 2x + y)$. Find the matrix of $T$ relative to the standard basis. *2,K3,CO4*
9. Find the singular values of the matrix $A = \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}$. *2,K3,CO5*
10. What is principle component analysis? *2,K1,CO5*

**PART - B (5 × 16 = 80 Marks)**

Answer ALL Questions

11. a) (i) Solve the following linear systems with Cramer's Rule *8,K3,CO1*
$x + 3y + 4z = 4, -x + 3y + 2z = 2, 3x + 9y + 6z = -6$.
(ii) Using properties of determinants, prove that *8, K3,CO1*
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$
OR
b) (i) Determine whether the following system has unique solution, *8, K3,CO1*

infinite number of solutions, or no solution: $x - 2y + 3z = 17$, $2x + y + 2z = 6$ and $2x - 4y + 6z = 34$.

(ii) Find the inverse of $A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 3 & 1 & 2 \\ 2 & 3 & 1 & 0 \\ 1 & 0 & 2 & 1 \end{pmatrix}$. *8, K3,CO1*

12. a) (i) Solve by Gauss elimination method, *8, K3,CO2*

$$x_1 + x_2 + x_3 + x_4 = 2, x_1 + x_2 + 3x_3 - 2x_4 = -6,$$
$$2x_1 + 3x_2 - x_3 + 2x_4 = 7,\ x_1 + 2x_2 + x_3 - x_4 = -2$$

(ii) Find the rank of a matrix $A = \begin{pmatrix} 1 & 2 & 6 & 6 \\ 4 & 7 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & 9 \end{pmatrix}$. *8, K3,CO2*

OR

b) Solve the below given system of equations by LU decomposition. *16, K3,CO2*
x + y + z = 1, 4x + 3y – z = 6, 3x + 5y + 3z = 4.

13. a) Verify the axioms of a vector space over the set $V$ of all ordered triples of real numbers of the form $(x, y, z)$ and defined operations $+$ and $\bullet$ by $(x, y, z) + (x', y', z') = (x + x', y + y', z + z')$ and $c(x, y, z) = (cx, y, z)$ . Is $V$ a vector space ? *16, K3,CO3*

OR

b) By using Gram Schmidt – orthogonalisation process, compute QR decomposition for $A = \begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$. *16, K3,CO3*

14. a) (i) Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$. *8, K3,CO4*

(ii) Find the linear transformation $T: V_3(R) \to V_3(R)$ determined by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ with the standard basis $\{e_1, e_2, e_3\}$. Hence find $T(-2, 2, 3)$. *8, K3,CO4*

OR

b) (i) Let $T: R^2 \to R^3$ is a linear transformation such that $(1, 1) = (1, 0, 2)$, $T(2, 3) = (1, -1, 4)$. Then find *8, K3,CO4*

1) T.
2) T(2, 5), T(8, 11).
3) Rank T.
4) Is T one-one or onto?

(ii) Show that the matrix $A = \begin{pmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{pmatrix}$ is unitary. Is A Hermitian? 8, K3CO4

15. a) Find the Singular value decomposition of the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix}$. 16, K3,CO5

OR

b) (i) Given the data in Table, reduce the dimension from 2 to 1 using the Principal Component Analysis (PCA) algorithm. 10, K3,CO5

| Feature | Example 1 | Example 2 | Example 3 | Example 4 |
|---|---|---|---|---|
| x | 4 | 8 | 13 | 7 |
| y | 11 | 4 | 5 | 14 |

(ii) Explain the application of Principal component analysis in image processing and machine learning. 6, K2,CO5