Reg. No.												

Question Paper Code	12009

17 JUL 2023

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL/MAY 2023

Fourth Semester

Electronics and Communication Engineering

(Common to Computer and Communication Engineering)

20BSMA401 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

(Regulations 2020)

(Use of Statistical Table is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

Marks, K-Level, CO

1. If X and Y are independent random variables with variance 2 and 3. Find the variance of $3X + 4Y$. *2,K2,CO1*
2. Find $E(X)$, if moment generating function of X is $\left(\frac{1}{4}+\frac{3}{4}e^t\right)^5$. *2,K2,CO1*
3. The joint probability mass function of a two-dimensional random variable (X, Y) is given by $P(x,y) = K(2x+3y), x = 1,2;\ y = 1,2$. Find the value of K. *2,K2,CO2*
4. Show that $f(x,y) = \frac{2}{5}(2x+3y),\ 0 \le x \le 1, 0 \le y \le 1$ is a joint pdf of X and Y. *2,K2,CO2*
5. Define a wide sense stationary process. *2,K1,CO3*
6. The autocorrelation function of a stationary random process is $R(\tau) = 16 + \frac{9}{1+16\tau^2}$. Find the mean and variance of the process. *2,K2,CO3*
7. Let A= $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ be a stochastic matrix. Check whether it is regular. *2,K1,CO4*
8. State any two properties of Poisson process. *2,K1,CO4*
9. Define memory less system. *2,K1,CO5*
10. Prove that the system is a linear time invariant system. *2,K2,CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) (i) A discrete random variable X has the following probability distribution: *16,K3,CO1*

X	0	1	2	3	4	5	6	7	8
$P(X)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

1) Find the value of 'a'.
2) Find $P(0 < X < 3)$, $P(X \leq 3)$.
3) Find the distribution function of X.

OR

b) Out of 800 families with 4 children each, how many families would be expected to have (a) 2 boys and 2 girls (b) atleast 1 boy (c) atmost 2 girls and (d) children of both genders. Assume equal probabilities for boys and girls. *16,K3,CO1*

12. a) The joint pdf is given by $f(x,y) = \frac{1}{3}(x+y), 0 \leq x \leq 1, 0 \leq y \leq 2$. Find a) the correlation coefficient of X, Y. b) Equation of regression lines. c) two regression curves for mean. *16,K3,CO2*

OR

b) (i) If X and Y are independent RVs with pdf $e^{-x}; x \geq 0$ and $e^{-y}; y \geq 0$, respectively, find the density function of $U = \frac{X}{X+Y}, V = X + Y$. Are U and V independent? *8,K3,CO2*

(ii) A lifetime of a certain brand of an electric bulb may be considered as a RV with mean 1200 h and standard deviation 250 h. Find the probability, using central limit theorem that the average lifetime of 60 bulbs exceed 1250 h. *8,K3,CO2*

13. a) The process $\{X(t)\}$ whose probability distribution under certain condition is given by $[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3,.. \\ \frac{at}{1+at}, & n = 0 \end{cases}$. Show that $\{X(t)\}$ is not stationary. *16,K3,CO3*

OR

b) (i) Verify whether the random process $\{X(t)\} = y\sin(\omega t + \theta)$is a wide sense stationary or not where y is uniformly distributed random variable in$(-1,1)$. *8,K3,CO3*

(ii) A random process $X(t) = A\cos at + B\sin at$, where A and B are independent random variables with mean zero and variances σ^2. Find the power spectral density of the process. *8,K3,CO3*

14. a) (i) If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 3$ and $C(t_1, t_2) = 4e^{-0.2|t_1 - t_2|}$. Find (1) $P(X(5) \leq 2)$ (2) $P(|X(8) - X(5)| \leq 1)$. *8,K3,CO4*

(ii) Three boys A, B, C are throwing a ball to each other. 'A' always throw the ball to B and B always throws to C but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. *8,K3,CO4*

OR

b) The probability matrix of a Markov chain $\{X_n\}$, $n = 1,2,3....$ having three states 1, 2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.4 & 0.3 & 0.3 \end{pmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$. *16,K3,CO4*

Find (1) $P(X_2 = 3, X_1 = 3, X_0 = 2)$.

(2) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (3) $P(X_2 = 3)$.

15. a) (i) If the input to a time-invariant, stable, linear system is WSS process, prove that the output will also be a WSS process. *8,K3,CO5*

(ii) A random process $X(t)$ with $R_{XX}(\tau) = e^{-\alpha|\tau|}$, α is a positive constant, is applied to a input of the linear system whose impulse response is $h(t) = e^{-bt}u(t)$, b is real constant. Find auto correlation of the output process $Y(t)$. *8,K3,CO5*

OR

b) A random process $X(t)$ with $R_{XX}(\tau) = e^{-2|\tau|}$ is the output to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \geq 0$. *16,K3,CO5*

Find a) cross correlation $R_{XY}(\tau)$

b) Cross spectral density $S_{XY}(\omega)$

c) Power spectral density $S_{XX}(\omega)$ between the input process $X(t)$ and the output process Y(t).