Reg. No.												

Question Paper Code	12532

B.E. / B.Tech - DEGREE EXAMINATIONS, NOV / DEC 2023

First Semester

(Common to All Branches except Computer Science and Business Systems)

20BSMA101 - ENGINEERING MATHEMATICS - I

(Regulations 2020)

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

Marks, K-Level, CO

1. Find the sum and product of all Eigen values of the matrix of *2,K1,CO1*
$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$
2. Use Cayley Hamilton theorem to find A^3 given that $A = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$. *2,K1,CO1*
3. Evaluate $\lim_{x\to 1}\left(\frac{x-1}{x^2-1}\right)$. *2,K2,CO2*
4. If $u = x + y, y = uv$, find the $\frac{\partial(x,y)}{\partial(u,v)}$. *2,K2,CO2*
5. Evaluate $\int \sqrt{1 + sin2x}\, dx$. *2,K1,CO3*
6. Evaluate $\int_0^{\frac{\pi}{2}} sin^6 x\, dx$. *2,K1,CO3*
7. Evaluate: $\int_0^{\pi/2}\int_0^{\sin\theta} r dr d\theta$. *2,K1,CO4*
8. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz dx dy dz$. *2,K2,CO4*
9. Define power series. *2,K2,CO5*
10. Define the Half range cosine series in *(0, l)*. *2,K2,CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) Reduce the quadratic form *16,K2,CO1*
$$6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$$
into a canonical form by an orthogonal transformation. Hence find rank, index, signature and nature of the quadratic form.

OR

b) Verify Cayley-Hamilton theorem for the matrix$= \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$. Also find A^{-1} and A^4 *16,K2,CO1*

12. a) A rectangular box open at the top is to have volume of 32cc. Find the dimension of the box that requires least material for its construction. *16,K3,CO2*

OR

b) (i) Find the Taylor's series expansion of $e^x siny\ at\ \left(-1,\frac{\pi}{4}\right)$ up to 2nd degree. *8,K3,CO2*

(ii) Find the absolute maximum and minimum values of *8,K3,CO2*

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1,\ [-2,3].$$

13. a) Prove that the reduction formula for $I_n = \int \sin^n x\,dx$ is $I_n = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}$. Hence find $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$. *16,K3,CO3*

OR

b) (i) The arc of the cardioid: $r = a(1 + cos\,\theta)$ included between $\theta = -\frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$ is rotated about the line$\theta = \frac{\pi}{2}$. Find the volume of the solid of revolution. *8,K3,CO3*

(ii) Evaluate$\int \frac{x^3}{(x-1)(x-2)} dx$. *8,K3,CO3*

14. a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals. *16,K4,CO4*

OR

b) (i) Evaluate by changing the order of integration in$\int_0^1 \int_{x^2}^{2-x} xydydx$. *12,K4,CO4*

(ii) Find the area of a circle of radius a by double integration. *4,K4,CO4*

15. a) Find the Fourier series for $f(x) = x^2$ in $-\pi \le x \le \pi$ and deduce that *16,K4,CO5*

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$

OR

b) Expand $\mathrm{f}(x) = \pi x - x^2$ **,** $\mathbf{0} < x < \boldsymbol{\pi}$ as a Fourier cosine series and deduce hence find $\sum_1^\infty \frac{\mathbf{1}}{\mathbf{n^4}} = \frac{\boldsymbol{\pi}^4}{\mathbf{90}}$**.** *16,K4,CO5*