Reg. No.																

Question Paper Code	12930

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2024

First Semester

Civil Engineering

(Common to All Branches)

20BSMA101 - ENGINEERING MATHEMATICS - I

Regulations - 2020

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

		Marks	*K – Level*	*CO*
1.	The product of two eigen values of the matrix A = $\begin{bmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ is 16. Find the third Eigen value of A.	*2*	*K1*	*CO1*
2.	State Cayley - Hamilton Theorem.	*2*	*K1*	*CO1*
3.	State Sandwich Theorem.	*2*	*K2*	*CO2*
4.	Evaluate $\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.	*2*	*K2*	*CO2*
5.	Evaluate $\int_0^{\frac{\pi}{2}} sin^6\, x\, dx.$	*2*	*K2*	*CO3*
6.	Evaluate $\int \frac{1}{(1+x^2)tan^{-1}x} dx.$	*2*	*K2*	*CO3*
7.	Change the order of integration in $\int_0^a \int_0^x f(x,y)dydx.$	*2*	*K2*	*CO4*
8.	Evaluate $\int_0^a \int_0^b \int_0^c (x^2+y^2+z^2)\, dxdydz.$	*2*	*K2*	*CO4*
9.	Define convergence series with an example.	*2*	*K1*	*CO5*
10.	State Dirichlet's conditions for a given function to expand in Fourier series.	*2*	*K1*	*CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

			Marks	*K – Level*	*CO*
11.	a) i)	Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$.	*8*	*K3*	*CO1*
	ii)	Use Cayley-Hamilton theorem to find the matrix $A^8-5A^7+7A^6-3A^5+8A^4-5A^3+8A^2-2A+I$ if the matrix A=$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.	*8*	*K3*	*CO1*
		OR			
	b)	Reduce the quadratic form $6x^2+3y^2+3z^2-4xy-2yz+4xz$ into a canonical form by an orthogonal transformation. Hence find rank,	*16*	*K3*	*CO1*

index, signature and nature of the quadratic form.

12. a) i) Find the values of a and b, — 8 K3 CO2

If $f(x) = \begin{cases} 2x-2 & if x < -1 \\ ax+b & if -1 \le x \le 1 \\ 5x+7 & if x \ge 1 \end{cases}$ is continuous for all real$x$.

ii) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. — 8 K3 CO2

OR

b) i) Expand $e^x cosy$ in powers of x and y upto terms of third degree. — 8 K3 CO2

ii) A rectangular box, open at the top, is to have a volume of 32cc. Find dimensions of box which least amount of material for its construction. — 8 K3 CO2

13. a) i) Evaluate $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\, dx$. — 8 K3 CO3

ii) Evaluate $\int \frac{1}{x^2\sqrt{x^2-1}} dx$. — 8 K3 CO3

OR

b) Prove that the reduction formula for $I_n = \int \cos^n x\, dx$ is — 16 K3 CO3

$I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$. Hence find $\int_0^{\frac{\pi}{2}} \cos^n x\, dx$.

14. a) i) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ by changing the order of integration. — 8 K3 CO4

ii) Find the volume of sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals. — 8 K3 CO4

OR

b) i) Evaluate$\iint_A r^3 drd\theta$, where A is the area between the circles $r = 2cos\theta$ and $r = 4cos\theta$. — 6 K3 CO4

ii) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing into polar coordinates and hence evaluate$\int_0^\infty e^{-x^2} dx$. — 10 K3 CO4

15. a) i) Find the radius of convergence and interval convergence of series $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$. — 6 K3 CO5

ii) Find the cosine series for $f(x) = x$ in $(0, \pi)$ and deduce — 10 K3 CO5

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{96}.$$

OR

b) Find the Fourier series for $f(x) = x^2$ in $-\pi \le x \le \pi$ and deduce that — 16 K3 CO5

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$