Reg. No.																

Question Paper Code	12931

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2024

First Semester

Computer Science and Business Systems

20BSMA102 - DISCRETE MATHEMATICS

Regulations - 2020

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

	Question	*Marks*	*K – Level*	*CO*
1.	Write the symbolic form of "If you work hard, you will be rewarded".	*2*	*K1*	*CO1*
2.	Construct the truth table for $(P \rightarrow Q) \vee (\neg P \rightarrow \neg Q)$.	*2*	*K2*	*CO1*
3.	Find the recurrence relation from $a_n - 2a_{n-1} = 3^n,\ a_1 = 5$.	*2*	*K2*	*CO2*
4.	State Pigeonhole Principle.	*2*	*K1*	*CO2*
5.	Prove that in a Boolean algebra, the complement of every element is unique.	*2*	*K2*	*CO3*
6.	Simplify the Boolean expression $a.c + c + [(b + b') + c]$.	*2*	*K1*	*CO3*
7.	Define adjacency matrix of simple graph.	*2*	*K1*	*CO4*
8.	Define planar graphs.	*2*	*K1*	*CO4*
9.	Give an example of a semi group but not a monoid.	*2*	*K1*	*CO5*
10.	Define a Ring.	*2*	*K1*	*CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

		Question	*Marks*	*K – Level*	*CO*
11.	a) i)	Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.	*8*	*K3*	*CO1*
	ii)	Find the PCNF of $(P \vee R) \wedge (P \vee \neg Q)$ without using truth table.	*8*	*K3*	*CO1*
		OR			
	b) i)	Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S), \neg R \vee P$ and Q.	*8*	*K3*	*CO1*
	ii)	Verify the validity of the following argument: Every living thing is a plant or an animal. Rama's dog is alive and it is not a plant. All animals have hearts. Therefore, Rama's dog has a heart.	*8*	*K3*	*CO1*
12.	a) i)	Prove by using Mathematical induction $3^n + 7^n - 2$ is divisible by 8, for $n \geq 1$.	*8*	*K3*	*CO2*
	ii)	Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n,\ n \geq 2$ given that $a_0 = 2$ and $a_1 = 8$.	*8*	*K3*	*CO2*

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create

OR

b) i) Solve the recurrence relation $a_{n+2} + 4a_{n+1} + 4a_n = 7$, $n \geq 0$ Given that $a_0 = 1$ and $a_1 = 2$. *8 K3 CO2*

ii) Use Mathematical induction show that $\sum_{k=1}^{n} k^2 = \frac{n\,(n+1)(2n+1)}{6}$. *8 K3 CO2*

13. a) i) In any Boolean algebra, prove that the following statements are equivalent (1) a + b = b (2) $a' + b = 1$ and (3) $a.b' = 0$. *8 K3 CO3*

ii) Minimize the function $f(a,b,c,d) = \Sigma(0,1,2,3,4,6,7,8,9,11,15)$ using Karnaugh map method. *8 K3 CO3*

OR

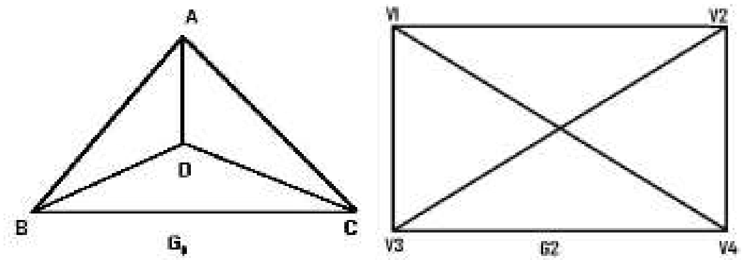
b) In a Boolean algebra, prove that $(a \wedge b)' = a' \vee b'$ and $(a \vee b)' = a' \wedge b'$. *16 K3 CO3*

14. a) i) State and prove the handshaking theorem. Also prove that the number of odd vertices in any graph is even. *10 K3 CO4*

ii) Prove that a tree with n vertices has n-1 edges. *6 K3 CO4*

OR

b) i) Determine whether the following graphs G_1 and G_2 are isomorphic *8 K3 CO4*



ii) Give an example of a graph which is *8 K3 CO4*

1. Eulerian but not Hamiltonian.
2. Hamiltonian but not Eulerian.
3. Both Eulerian and Hamiltonian.
4. Non Eulerian and non-Hamiltonian.

15. a) i) Show that $(Q^+, *)$ is an abelian group, where $*$ is defined by *10 K3 CO5*

$$a * b = \frac{ab}{2}, \forall\, a, b \in Q^+.$$

ii) If $f: G \rightarrow G'$ is a group homomorphism from $\{G, *\}$ to $\{G', \Delta\}$ then prove that for any $a \in G$, $f(a^{-1}) = [f(a)]^{-1}$. *6 K3 CO5*

OR

b) State and prove Lagrange's theorem. *16 K3 CO5*