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Question Paper Code	12538
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B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2023
 Second Semester
Computer Science and Business Systems
20BSMA202 - LINEAR ALGEBRA
 (Regulations 2020)

Duration: 3 Hours

Max. Marks: 100

PART-A (10 × 2 = 20 Marks)
 Answer ALL Questions

*Marks,
K-Level, CO*

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| 1. | If $A + B = \begin{pmatrix} 1 & -1 \\ 3 & 0 \end{pmatrix}$ and $A - B = \begin{pmatrix} 3 & 1 \\ 1 & 4 \end{pmatrix}$, calculate the product AB . | 2,K2,CO1 |
| 2. | Define Inverse of a matrix. | 2,K1,CO1 |
| 3. | If the rank of the matrix $\begin{pmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 3 & 5 & k \end{pmatrix}$ is 2, find the value of k . | 2,K2,CO2 |
| 4. | When does the Gauss elimination method fail? | 2,K2,CO2 |
| 5. | State Dimension theorem. | 2,K1,CO3 |
| 6. | Find the linear span of $S = \{(1,0,0), (2,0,0), (3,0,0)\} \subset R^3$ | 2,K2,CO3 |
| 7. | Define Linear transformations. | 2,K1,CO4 |
| 8. | Define Hermitian Matrix. | 2,K1,CO4 |
| 9. | Write down singular value decomposition of a square matrix. | 2,K1,CO5 |
| 10. | Explain Principal Component Analysis with an example. | 2,K2,CO5 |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

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| 11. | a) | (i) Prove that $\begin{vmatrix} a - b - c & 2b & 2c \\ 2a & b - c - a & 2c \\ 2a & 2b & c - a - b \end{vmatrix} = (a + b + c)^3$. | 8,K3,CO1 |
| | | (ii) Solve the equations $3x + y + 2z = 3$; $2x - 3y - z = -3$;
$x + 2y + z = 4$ by using Cramer's rule. | 8,K3,CO1 |

OR

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|----|--|-----------|
| b) | Solve the following equations by calculating the inverse by elementary row operations: | 16,K3,CO1 |
| | $2x_1 + 2x_2 + 2x_3 - 3x_4 = 2$; $3x_1 + 6x_2 - 2x_3 + x_4 = 8$; | |
| | $x_1 + x_2 - 3x_3 - 4x_4 = -1$; $2x_1 + x_2 + 5x_3 + x_4 = 5$ | |

12. a) (i) Find the rank of $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{pmatrix}$. 8,K3,CO2
- (ii) Solve the following equations by Gauss- Elimination method 8,K3,CO2
- $$\begin{aligned} 27x + 6y - z &= 85 \\ x + y + 54z &= 110 \\ 6x + 15y + 2z &= 72 \end{aligned}$$

OR

- b) Solve the system of linear equations by LU decomposition method: 16,K3,CO2
- $$2x + 3y - z = 5; \quad 4x + 4y - 3z = 3; \quad 2x - 3y + 2z = 2.$$
13. a) (i) Verify whether the set V of all ordered triples of real numbers of the form $(x, y, 0)$ and defined operations $+$ and \bullet by $(x, y, 0) + (x', y', 0) = (x + x', y + y', 0)$ and $c(x, y, 0) = (cx, cy, 0)$ is a vector space or not. 8,K3,CO3
- (ii) Check whether the set 8,K3,CO3
- $$S = \{-1 - x + 2x^2, 2 + x - 2x^2, 1 - 2x + 4x^2\}$$
- forms a basis for
- $P_2(\mathbb{R})$
- ?

OR

- b) Construct a QR-decomposition for the matrix $A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$. 16,K3,CO3

14. a) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $T(x, y) = (x - y, x, 2x + y)$. 16,K3,CO4
- Let $B = \{e_1, e_2\}, e_1 = (1, 0), e_2 = (0, 1)$ be the basis for \mathbb{R}^2 and $B' = \{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$ be a basis for \mathbb{R}^3 . Find the matrix of T in these bases.

OR

- b) Find the eigen values and eigen vectors on ordered basis β for V such that $[T]_{\beta}$ is a diagonal matrix where T be the linear operator on $V = \mathbb{R}^3$ and $T(a, b, c) = (7a - 4b + 10c, 4a - 3b + 8c, -2a + b - 2c)$. Is T diagonalizable? 16,K3,CO4

15. a) Construct a singular value decomposition for the matrix 16,K3,CO5
- $$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & 2 & 6 \end{pmatrix}$$

OR

- b) Find the principal components for the following data: 16,K3,CO5

Features	A	B	C	D
X_1	6	9	15	7
X_2	2	11	7	16