Reg. No.															

Question Paper Code	13326

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2024

Second Semester

Computer Science and Business Systems

20BSMA202 - LINEAR ALGEBRA

Regulations - 2020

Duration: 3 Hours Max. Marks: 100

PART - A (MCQ) (20 × 1 = 20 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
1.	Find the value of k, for which $\begin{pmatrix} k & 8 \\ 4 & 2k \end{pmatrix}$ is a singular matrix (a) 4 (b) -4 (c) ± 4 (d) 0	1	K2	CO1
2.	What is the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$ (a) 2 (b) 1 (c) 0 (d) 3	1	K2	CO1
3.	When is the Cramer's rule applicable for a system m linear equations and n variables (a) m = n, co-efficient matrix is non-singular (b) m ≠ n only (c) m ≠ n and coefficient matrix is singular (d) m = n only	1	K1	CO1
4.	For what values of α and β, the system of equation $4x + y = \alpha$ and $\beta x + 2y = 3$ has no solution (a) $\alpha \neq \frac{3}{8}$ & β = 8 (b) α =1 & β = 8 (c) α= 1 & β = 3 (d) α = 8 & $\beta \neq \frac{3}{8}$	1	K1	CO1
5.	If A is the Co-efficient matrix, K is the augmented matrix and R is the rank of the matrix, then (a) If R(A) ≠ R(K), the equations are inconsistent and have no solution (b) If R(A) = R(K) = n, the equations are consistent and have unique solution (c) If R(A) = R(K) < n, the equations are consistent and have infinite solutions (d) If R(A) = R(K) > n, the equations are consistent and have infinite solutions	1	K1	CO2
6.	A system of equations is said to be inconsistent if (a) they have one solution (b) they have no solutions (c) they have one or more solutions (d) none of these	1	K1	CO2
7.	The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ (a) 2 (b) 0 (c) 3 (d) a + b + c	1	K2	CO2
8.	Gauss elimination method is used to find (a) Rank of the matrix (b) Determinant of the matrix (c) Inverse of the invertible matrix (d) All of the above	1	K1	CO2
9.	The additive identity is (a) 1 (b) 0 (c)-1 (d) none of the above	1	K1	CO3
10.	Intersection of two subspaces of a vector space V(F) is a subspace of V(F) (a) True (b) False (c) Partially true (d) none of the above	1	K1	CO3
11.	The range of the linear transformation T is also called as (a) Kernel (b) Image (c) Null space (d) none of the above	1	K1	CO3
12.	The matrix Q in QR decomposition is (a) Upper triangular matrix (b) Lower triangular matrix (c) Orthonormal matrix (d) None of the above	1	K1	CO3
13.	A map T:V→W defined by T(v)=0 for every $v \in V$ is called as (a) Identity transformation (b) zero transformation (c) Isomorphic (d) none of the above	1	K1	CO4

14.	Product of the eigen values of a square matrix A is equal to (a)Trace of the matrix (b) $\|A\|$ (c) $\|A^T\|$ (d) None of the above	1	K1	CO4
15.	The positive definite matrix is (a) Symmetric (b) Skew-symmetric (c) Both (d) None of the above	1	K1	CO4
16.	Dimension theorem states that (a) Rank(T) - nullity(T) = dimV (b) Rank(T) + nullity(T) = dimW (c) Rank(T) + nullity(T) = dimV (d) None of the above	1	K1	CO4
17.	Which of the following techniques would perform better for reducing dimension? (a) Removing columns which have too many missing values (b) Removing columns which have high variance in data (c) Removing columns with dissimilar data trends (d) None of these	1	K1	CO5
18.	In SVD of a matrix A, the eigenvalues are calculated for (a) A matrix (b) A^T matrix (c) AA^T matrix (d) None of the above	1	K1	CO5
19.	The mathematical technique used in image processing is (a) Probability and Statistics (b) Difference equation (c) Fourier transforms (d) All of the above	1	K1	CO5
20.	Singular Value decomposition in machine learning is a versatile tool for (a) dimensionality reduction (b) data compression (c) feature extraction (d) All of the above	1	K1	CO5

PART - B (10 × 2 = 20 Marks)

Answer ALL Questions

21.	Without expanding the determinant, prove that $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0.$	2	K2	CO1
22.	Find the inverse of the matrix $\begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix}$.	2	K2	CO1
23.	Write the condition for infinite solutions for the system of linear equations.	2	K1	CO2
24.	In R^3 over R,test whether (2,-5,4) is a linear combination of the vectors (1,-3,2) and (2,-1,1).	2	K2	CO2
25.	State and prove Cancellation law of addition.	2	K2	CO3
26.	Write the basis for the set of all matrices M2 × 2 (R).	2	K1	CO3
27.	Define Range and Null space.	2	K1	CO4
28.	Test whether the map T: $R^3 \to R^3$ defined by $T(x, y, z) = (x, 0, 0)$ for every $(x, y, z) \in R$ is a linear.	2	K2	CO4
29.	What is the advantage of rank in machine learning problems?	2	K1	CO5
30.	Write the formula for sample principal component.	2	K1	CO5

PART - C (6 × 10 = 60 Marks)

Answer ALL Questions

31.	a)	Solve by matrix method, the system of equations $x + y + z = 1$, $3x + y - 3z = 5$, $x - 2y - 5z = 0$.	10	K3	CO1
		OR			
	b)	Show that $\begin{vmatrix} a-b-c & 2b & 2c \\ 2a & b-c-a & 2c \\ 2a & 2b & c-a-b \end{vmatrix} = (a+b+c)^3$.	10	K3	CO1
32.	a)	Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them by rank method.	10	K3	CO2
		OR			

			Marks	K	CO
	b)	Solve the following system of equations by triangularization: $4x + 5y + 2 = 4$; $2x - 4y - 2z = -4$; $-5x + 3y - 3z = -23$.	10	K3	CO2
33.	a)	Test whether the polynomial $2x^3 - x^2 + x + 3$ is in the linear span of $S = \{x^3 + x^2 + x + 1,\ \ x^2 + x + 1, x + 1\}$.	10	K3	CO3
		OR			
	b)	Let V be an inner product space and $S = \{v_1, v_2, \ldots, v_n\}$ be an orthogonal subset of V consisting of non-zero vectors. If v ∈ L(S), then prove that $V = \sum_{i=1}^{n} \frac{\langle v, v_i \rangle}{\|\|v_i{}^2\|\|}\ v_i$.	10	K3	CO3
34.	a)	Find the matrix of T in the standard basis for the transformation T : $P_2(R) \to P_2(R)$ where $T(f(x)) \ = f^{'}(x)$.	10	K3	CO4
		OR			
	b)	Find the linear transformation by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ with respect to the standard bases, what is $T(-2, 2, 3)$?	10	K3	CO4
35.	a)	Determine the population principal components Y_1 and Y_2 for the covariance matrix $A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$. Also calculate the proportion of the total population variance explained by first principal component.	10	K3	CO5
		OR			
	b)	Suppose A_0 has these two measurements of 5 samples. A_0=$\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix}$ Compute the centred matrix A, sample covariance S, eigen values$\lambda_{1,}\lambda_2$. what is the line through the origin is closest to the 5 sample in the column of A.	10	K3	CO5
36.	a) i)	Verify whether V=$\{(a_1, a_2) : a_1, a_2 \in R\}$ under component wise addition and scalar multiplication given by α$(a_1,\ a_2) = (\ a_1, 0)$ is a vector space or not.	5	K3	CO3
	ii)	Let T: $P_2(R) \to P_2(R)$ defined by $T[f(x)] = f(x) + (x + 1)f^{'}(x)$ and $B = \{1,\ x,\ x^2\}$ be an ordered basis of $P_2(R)$with $A = [T]_B$. 1. Find the matrix A.	5	K3	CO4
		OR			
	b) i)	Determine whether the set $S = \{(1, 2, 3), (4, 1, 5), (-4, 6, 2)\}$ of vectors in $R^3(R)$ is linearly dependent or independent.	5	K3	CO3
	ii)	Let T: V → W be a linear transformation. Then T is one to one if N (T) = {0}.	5	K3	CO4