Reg. No.																

Question Paper Code	13327

B.E. / B.Tech. / M.Tech - DEGREE EXAMINATIONS, NOV / DEC 2024

Second Semester

Computer Science and Engineering

(Common to Information Technology, Computer Science and Engineering (AIML) Computer Science and Engineering (IOT), Artificial Intelligence and Data Science & M.Tech. - Computer Science and Engineering (5 years Integrated))

20BSMA204 - DISCRETE STRUCTURES

Regulations - 2020

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (20 × 1 = 20 Marks)

Answer ALL Questions

	Question	Marks	K – Level	CO
1.	Which of the following relations is symmetric but neither reflexive nor transitive for a set A = {1, 2, 3}. (a) R = {(1, 2), (1, 3), (1, 4)} (b) R = {(1, 2), (2, 1)} (c) R = {(1, 1), (2, 2), (3, 3)} (d) R = {(1, 1), (1, 2), (2, 3)}	1	K2	CO1
2.	If A has m elements and B has n elements then number of relations from A to B is (a) m^n (b) n^m (c) mn (d) None of the above	1	K2	CO1
3.	If f:R→R, $g(x) = 3x^2 + 7$ and $f(x) = \sqrt{x}$, then gof(x) is equal to _______ (a) 3x-7 (b) 3x-9 (c) 3x+7 (d) 3x-8	1	K2	CO1
4.	A function is invertible if it is ____________ (a) surjective (b) bijective (c) injective (d) neither surjective nor injective	1	K1	CO1
5.	If m pigeons are assigned to n pigeonholes, then there must be a pigeonhole containing At least (a) $\left[\left(\frac{m-1}{n}\right)\right] + 1$ pigeons (b) $\left[\left(\frac{m-1}{n}\right)\right] - 1$ pigeons (c) $\left[\left(\frac{m+1}{n}\right)\right] + 1$ pigeons (d) $\left[\left(\frac{m-n}{n}\right)\right] + 1$ pigeons	1	K1	CO2
6.	Find the number of ways of arranging the letters of the words DANGER, so that no vowel occupies odd place. (a) 36 (b) 48 (c) 144 (d) 720	1	K2	CO2
7.	$2^n < n!$ is true for (a) $n \geq 1$ (b) $n \geq 2$ (c) $n \geq 3$ (d) $n \geq 4$	1	K1	CO2
8.	Let the players who play cricket be 12, the ones who play football 10, those who play only cricket are 6, then the number of players who play only football are ___________, assuming there is a total of 16 players. (a) 16 (b) 8 (c) 4 (d) 10	1	K2	CO2
9.	Contrapositive of p →q is the proposition ____? (a) ~p→~q (b) ~q→~p (c) q→~p (d) ~q→p	1	K1	CO3
10.	p → q is logically equivalent to ________ (a) ¬p ∨ ¬q (b) p ∨ ¬q (c) ¬p ∨ q (d) ¬p ∧ q	1	K1	CO3
11.	What rules of inference are used in this argument? "All students in this science class has taken a course in physics" and "Marry is a student in this class" imply the conclusion "Marry has taken a course in physics." (a) Universal Specification (b) Universal generalization (c) Existential Specification (d) Existential generalization	1	K2	CO3
12.	$(p \rightarrow q) \wedge (p \rightarrow r)$ is logically equivalent to (a) p → (q ∨ r) (b) p → (q ∧ r) (c) p ∧ (q → r) (d) p ∧ (q → r)	1	K1	CO3
13.	How many properties can be held by a group? (a) 2 (b) 3 (c) 5 (d) 4	1	K1	CO4

14. A non empty subset H of group G is a subgroup of G if: *1 K1 CO4*
(a) An identity element is a∈ H.
(b) The operation of G closes H, meaning that if a, b∈ H, then ab∈ H
(c) Inverses of H have closed forms, i.e., if a∈ H then a^{-1}∈ H.
(d) All of the above

15. The elements of the set $\{1, i, -i, -1\}$ forms a *1 K2 CO4*
(a) semigroup (b) subgroup (c) cyclic group (d) abelian group

16. The complement of 0 is 0' = _____. *1 K2 CO4*
(a) zero (b) 1 (c) -1 (d) ±1

17. Pendant vertex is a vertex with degree ____? *1 K1 CO5*
(a) Zero (b) One (c) Two (d) Three

18. How many edges are there in a complete graph with n vertices? *1 K1 CO5*
(a) n (n - 1) / 2 (b) n / 2 (c) n (n + 1) / 2 (d) n (n - 1)

19. As each of the m vertices is connected to each of the n vertices, a complete bipartite graph consists of ___ edges? *1 K1 CO5*
(a) m+n (b) m-n (c) m.n (d) m/n

20. An acyclic undirected graph is called ___________ *1 K1 CO5*
(a) Regular (b) Complete (c) Circuit (d) Tree

PART - B (10 × 2 = 20 Marks)
Answer ALL Questions

21. Let f : Z → Z be such that $f(x) = x + 1$. Is f invertible, and if it is, what is its inverse? *2 K2 CO1*
22. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Find $f \circ g$ and $g \circ f$. *2 K2 CO1*
23. If seven colors are used to paint 50 bicycles, then show that at least 8 bicycles will be the same color. *2 K2 CO2*
24. In how many ways can all the letters in MATHEMATICS be arranged? *2 K2 CO2*
25. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent. *2 K2 CO3*
26. Give an indirect proof of the theorem "If 3n+2 is odd, then n is odd". *2 K2 CO3*
27. Show that every element of a group G is self-inverse then G is abelian. *2 K2 CO4*
28. Show that $(Z_5, +_5)$ is a cyclic group. *2 K2 CO4*
29. How many edges are there in a graph with 10 vertices each of degree six? *2 K2 CO5*
30. Give an example of a graph which is both Eulerian and Hamiltonian. *2 K1 CO5*

PART - C (6 × 10 = 60 Marks)
Answer ALL Questions

31. a) Let m be an integer with $m > 1$. Show that the relation $R = \{(a, b) \mid a \equiv b \pmod{m}\}$ is an equivalence relation on the set of integers. *10 K3 CO1*

OR

b) If $f : R \rightarrow R$ *and* $g : R \rightarrow R$ be any two function, where R is the set of all real numbers. Find $f \circ g$ and $g \circ f$ where $f(x) = x^2 - 2, g(x) = x + 4$. State whether these functions are injective, surjective, and bijective. *10 K3 CO1*

32. a) Prove by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer. *10 K3 CO2*

OR

b) Find the numbers between 1 to 500 that are not divisible by any of the integers 2 or 3 or 5 or 7. *10 K3 CO2*

33. a) Without using truth table find PDNF, PCNF of $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$. *10 K3 CO3*

OR

b) Use rules of inferences to obtain the conclusion of the following arguments:” Babu is a student in this class, knows how to write programmes in JAVA”. “Everyone who knows how to write programmes in JAVA can get a high-paying job”. Therefore, “someone in this class can get a high-paying job”. *10 K3 CO3*

34. a) State and Prove Lagrange’s theorem on finite groups. *10 K3 CO4*

OR

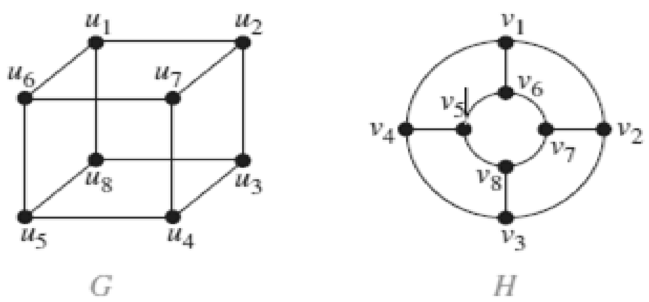
b) Prove that $G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ forms an abelian group under matrix multiplication. *10 K3 CO4*

35. a) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$ *10 K3 CO5*

OR

b) Prove that a connected graph G is Euler graph if and only if every vertex of G is of even degree. *10 K3 CO5*

36. a) i) Determine whether the graphs are isomorphic are not *5 K3 CO5*



ii) In any group $\langle G, * \rangle$, show that $(a*b)^{-1} = b^{-1}a^{-1}$, for all $a, b \in G$ *5 K3 CO4*

OR

b) i) If $*$ is the operation defined on $S = Q \times Q$ where Q is the set of all rational numbers and given by $(a, b) * (x, y) = (ax,\ ay + b)$, then show that $(S, *)$is a semigroup. Is it commutative? *5 K3 CO4*

ii) Which of the simple graphs given have a Hamilton circuit or if not a Hamilton path *5 K3 CO5*