Question Paper Code	12731

B.E. / B.Tech. / M. Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2024

Second Semester

Computer Science and Engineering

(Common to Computer Science and Engineering (AIML), Computer Science and Engineering (IoT), Computer Science and Engineering (CS), Artificial Intelligence and Data Science, Information Technology & M.Tech - Computer Science and Engineering)

20BSMA204 - DISCRETE STRUCTURES

Regulations - 2020

Duration: 3 Hours   Max. Marks: 100

PART - A (10 × 2 = 20 Marks)
Answer ALL Questions

		Marks	K–Level	CO
1.	Define partial ordered relation.	2	K1	CO1
2.	Let $f : R \to R$ be given by $f(x) = x^3 - 2$, find f^{-1}.	2	K2	CO1
3.	How many permutations are there on the word 'MATHEMATICS'?	2	K2	CO2
4.	State the Pigeonhole principle.	2	K1	CO2
5.	Write the contra positive of "If there is rain then I buy an umbrella'.	2	K2	CO3
6.	Express the statement "Good food is not cheap" in symbolic form.	2	K2	CO3
7.	Define subgroup.	2	K1	CO4
8.	Define cyclic group.	2	K1	CO4
9.	State Hand shaking theorem.	2	K1	CO5
10.	Define a path in a graph.	2	K1	CO5

PART - B (5 × 16 = 80 Marks)
Answer ALL Questions

		Marks	K–Level	CO
11. a) i)	Let Z be the set of integers. Define a relation R on Z by $R = \{(x, y) \in Z \times Z : x - y \text{ is divisible by } 3\}$ Then show that R is an equivalence relation.	8	K3	CO1
ii)	Determine which of the following functions are bijective, where $f : R \to R$, defined by (i) $f(x) = 3x^3 + x$; (ii) $f(x) = -4x^2 + 12x - 9$.	8	K3	CO1

OR

		Marks	K–Level	CO
b) i)	Prove that the inverse of a function f is unique if it exists.	8	K3	CO1
ii)	Let $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ for $x \in R$. where R is the set of all real numbers. Find $g \circ f$, $f \circ g$, $f \circ f$, $g \circ g$, $f \circ h$, $h \circ g$, $h \circ f$, $f \circ h \circ g$.	8	K3	CO1

		Marks	K–Level	CO
12. a) i)	Prove by mathematical induction, $n^3 + 2n$ is divisible by 3 for $n \geq 1$.	8	K3	CO2
ii)	How many permutations can be made out of the letters of the word "COMPUTER"?How many of these (i) begin with C (ii) end with R (iii) Begin with C and End with R (iv) C and R occupy the end places.	8	K3	CO2

OR

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create

b) i) Prove by mathematical induction $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$.   8   K3   CO2

ii) Find the number of integers between 1 to 100 that are divisible by the integers 2,3,5 or 7   8   K3   CO2

13. a) i) Prove that the premises   8   K3   CO3

$P \to Q, \ Q \to R, \ R \to S, \ S \to \neg R \ \ and \ \ P \wedge S$ are inconsistent.

ii) Using the rule CP or otherwise show the following implications.   8   K3   CO3

$$(\ni x)P(x) \to (x)Q(x) \Rightarrow (x)\big(P(x) \to Q(x)\big)$$

OR

b) i) Show that $(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.   8   K3   CO3

ii) Show that the premises "one student in this class knows how to write programs in JAVA" and "everyone who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "someone in this class can get a high-paying job".   8   K3   CO3

14. a) State and prove Lagrange's theorem in a group.   16   K3   CO4

OR

b) i) Let $G=\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}\right\}$, show that G is a group under the operation of matrix multiplication.   8   K3   CO4

ii) In any Boolean algebra, prove that the following statements are equivalent   8   K3   CO4

(i) $a + b = b$; (ii) $a.b = a$; (iii) $a' + b = 1$; (iv) $a.b' = 0$

15. a) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\dfrac{(n-k)(n-k+1)}{2}$.   16   K3   CO5

OR

b) i) Determine whether the graphs are isomorphic or not.   8   K3   CO5

G   H
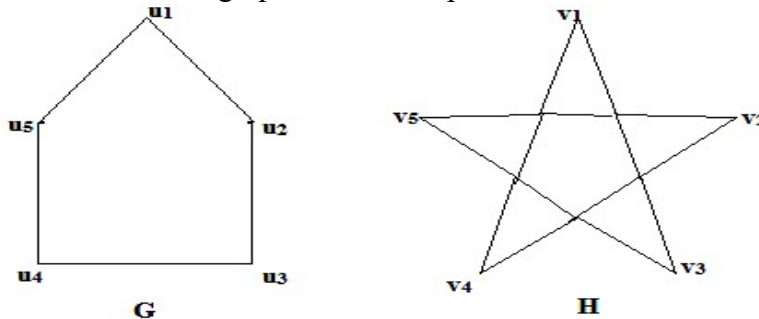
b) ii) Give an example of a graph which is   8   K3   CO5
  (1) Eulerian but not Hamiltonian
  (2) Hamiltonian but not Eulerian
  (3) Hamiltonian and Eulerian
  (4) Neither Hamiltonian nor Eulerian.