

Reg. No.																				
----------	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Question Paper Code	12902
---------------------	-------

**B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2024**

Third Semester

**Civil Engineering**

**(Common to Computer and Communication Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering & Instrumentation and Control Engineering)**

**20BSMA301 - LINEAR ALGEBRA, PARTIAL DIFFERENTIAL EQUATIONS AND TRANSFORMS**

Regulations - 2020

Duration: 3 Hours

Max. Marks: 100

**PART - A (10 × 2 = 20 Marks)**

Answer ALL Questions

	Marks	K- Level	CO
1. Define linear combination.	2	K1	CO1
2. Show that the vectors (1,2,3),(3,-2,1),(1,-6,-5) in $R^3$ are linearly dependent over $R$ .	2	K2	CO1
3. State Dimension theorem	2	K1	CO2
4. Is there a linear transformation $T : R^3 \rightarrow R^2$ such that $T(1,0,3)=(1,1)$ and $T(-2,0,-6)=(2,1)$ ? Justify.	2	K2	CO2
5. Solve $[D^2 - 4DD' + 3D'^2]z = 0$ .	2	K2	CO3
6. Form the PDE by eliminating the arbitrary constants a, b from $(x - a)^2 + (y - b)^2 + z^2 = 1$ .	2	K1	CO3
7. Find the Fourier sine transform of $f(x) = \frac{1}{x}$ .	2	K2	CO4
8. Define convolution theorem for Fourier transforms.	2	K1	CO4
9. Find $Z[n^2]$ .	2	K2	CO5
10. Form the difference equation from the relation $y_n = a + b \cdot 3^n$	2	K1	CO5

**PART - B (5 × 16 = 80 Marks)**

Answer ALL Questions

11. a) i) Show that $F^n = \{(a_1, a_2, \dots, a_n) : a_i \in F\}$ is a vector space over $F$ with respect to addition and scalar multiplication defined component wise.	10	K3	CO1
ii) Determine whether the set $W = \{(a_1, a_2) : 2a_1 + 3a_2 = 0 : a_1, a_2 \in R^2\}$ is subspace or not.	6	K3	CO1
<b>OR</b>			
b) i) Check whether $2x^3 - 2x^2 + 12x - 6$ is a linear combination of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$ .	8	K3	CO1
ii) Test whether the set of vectors $\{(1, -3, -2), (-3, 1, 3), (-2, -10, 2)\}$ in $R^3$ form a basis of $R^3$ over $R$ .	8	K3	CO1

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create

**12902**

12. a) i) If  $T:R^2 \rightarrow R^3$  is a linear transformation such that  $T(1,1) = (1,0,2)$ ,  $T(2,3) = (1,-4,4)$  8 K3 CO2

(a) Determine T.

(b)  $T(2,5), T(8,11)$ .

(c) Rank T.

(d) Is T one-to-one or onto?

- ii) Let  $V = P(R)$ , the vector space of polynomials over  $R$  with inner product defined by  $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ , where  $f(t) = t + 2$ ,  $g(t) = t^2 - 2t - 3$ . Find  $\|f\|$ ,  $\|f + g\|$  and  $\|g\|$ . 8 K3 CO2

**OR**

- b) Find an Orthonormal basis of the inner product space  $R^3(R)$  with the standard inner product, given the basis  $\{(1,0,1), (0,1,1), (1,3,3)\}$  using Gram-Schmidt process. Also find the Fourier coefficients of the vector  $(1,1,2)$  relative to the Orthonormal basis. 16 K3 CO2

13. a) i) Solve  $(mz - ny)p + (nx - lz)q = ly - mx$ . 8 K3 CO3

- ii) Solve  $z = px + qy + p^2 - q^2$ . 8 K3 CO3

**OR**

- b) i) Solve  $[D^2 - D'^2]z = e^{x+2y} + \sin(2x - y)$ . 8 K3 CO3

- ii) Solve  $(x^2 - yz)p + (y^2 - xz)q = (z^2 - xy)$ . 8 K3 CO3

14. a) Find the Fourier transform of the function  $f(x)$  defined by 16 K3 CO4

$$f(x) = \begin{cases} 1 - x^2 & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases} . \text{ Hence prove that}$$

$$\int_0^\infty \frac{\text{sins} - \text{scoss}}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$$

$$\text{Also show that } \int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)^2 dx = \frac{\pi}{15}.$$

**OR**

- b) i) Evaluate  $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$  using Fourier Transform. 8 K3 CO4

- ii) Evaluate  $\int_0^\infty \frac{x^2 dx}{(x^2+1)^2}$  using Parseval's identity. 8 K3 CO4

15. a) i) Find the inverse Z-transform of  $\frac{z^2 + z}{(z-1)(z^2 + 1)}$  using partial fraction. 8 K3 CO5
- ii) Using Z-transform, solve  $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$  given that  $y_0 = 0, y_1 = 0$  8 K3 CO5

**OR**

- b) i) Using convolution theorem, find inverse Z-transform of  $\frac{8z^2}{(2z-1)(4z-1)}$ . 8 K3 CO5
- ii) Find the Z transform of  $\left\{\frac{1}{(n+1)!}\right\}$  and  $\left\{\cos\frac{n\pi}{2}\right\}$ . 8 K3 CO5