

Reg. No.

Question Paper Code

12454

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2023

Third Semester

CIVIL ENGINEERING

(Common to Electronics and Communication Engineering, Electrical and Electronics Engineering, Electronics and Instrumentation Engineering, Instrumentation and Control Engineering & Computer and Communication Engineering)

20BSMA301 – LINEAR ALGEBRA, PARTIAL DIFFERENTIAL EQUATIONS AND TRANSFORMS

(Regulations 2020)

Duration: 3 Hours

Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

- | | <i>Marks,
K-Level, CO</i> |
|---|-------------------------------|
| 1. Show that the vectors $(1,2,3), (3,-2,1), (1,-6,-5)$ in R^3 are linearly dependent over R . | <i>2,K2,CO1</i> |
| 2. State the necessary and sufficient condition for a subset W to be subspace of a vector space V over F . | <i>2,K1,CO1</i> |
| 3. State Dimension theorem for Linear transformation. | <i>2,K1,CO2</i> |
| 4. Let $x = (2, 1 + i, i)$ and $y = (2 - i, 2, 1 + 2i)$ be vectors in C^3 . Compute $\langle x, y \rangle$. | <i>2,K2,CO2</i> |
| 5. Solve $[D + D' - 1][D - 2D' + 3]z = 0$. | <i>2,K2,CO3</i> |
| 6. Solve $\sqrt{p} + \sqrt{q} = 1$. | <i>2,K2,CO3</i> |
| 7. Define Fourier sine transform pair. | <i>2,K1,CO4</i> |
| 8. If $F(s)$ is the Fourier Transform of $F(x)$, prove that $F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right), a \neq 0$. | <i>2,K2,CO4</i> |
| 9. Find the Z-transform of $\frac{1}{n!}$. | <i>2,K2,CO5</i> |
| 10. Form a difference equation from $y_n = A \cdot 3^n$. | <i>2,K2,CO5</i> |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) Prove that the set of all $m \times n$ matrices over F denoted by $M_{m \times n}(F)$ is a vector space over F with respect to matrix addition and scalar multiplication. *16,K3,CO1*

OR

b) (i) Determine if the set $\{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\}$ is linearly dependent or linearly independent in $P_3(R)$. 8,K3,CO1

(ii) Determine whether the given vector $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, is in the span of S , where $S = \left\{ \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\}$. 8,K3,CO1

12. a) Find an orthogonal basis of the inner product space $R^3(R)$ with standard inner product, given the basis $B = \{(1, 1, 0), (1, -1, 1), (-1, 1, 2)\}$ using Gram-Schmidt orthogonalization process. Also find the Fourier coefficients of the vector $(2, 1, 3)$ relative to orthonormal basis. 16,K3,CO2

OR

b) (i) If $T: R^2 \rightarrow R^3$ is a linear transformation such that $T(1, 1) = (1, 0, 2)$, $T(2, 3) = (1, -4, 4)$ 10,K3,CO2

1. Determine T
2. $T(2, 5)$ & $T(8, 11)$
3. Rank T
4. Is T one-to-one?

(ii) Let $V = P(R)$, the vector space of polynomials over R with inner product defined by $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$, where $f(t) = t + 2$, $g(t) = t^2 - 2t - 3$. Find $\langle f, g \rangle$, $\|f\|$ and $\|g\|$. 6,K3,CO2

13. a) (i) Solve $(3z - 4y)p + (4x - 2z)q = 2y - 3x$. 8,K3,CO3

(ii) Form the partial differential equation by eliminating f from $f(x^2 + y^2 + z^2, x + y + z) = 0$. 8,K3,CO3

OR

b) (i) Solve $[D^3 - 7DD^2 - 6D^3]z = \sin(x + 2y) + e^{2x+y}$. 8,K3,CO3

(ii) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$. 8,K3,CO3

14. a) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| \geq a \end{cases}$. Hence deduce that $\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$ and $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. 16,K3,CO4

OR

b) (i) Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)^2}$ using Parseval's identity. 8,K3,CO4

(ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier transforms. 8,K3,CO4

15. a) (i) Find the inverse Z-transform of $\frac{10z}{z^2-3z+2}$ using partial fraction method. 8,K3,CO5

(ii) Using Z-transform, solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that $y_0 = 0, y_1 = 0$. 8,K3,CO5

OR

b) (i) Using convolution theorem, find inverse Z - transform of $\frac{z^2}{(z-a)(z-b)}$. 8,K3,CO5

(ii) Find the Z-transform of $a^n \sin n\theta$ and $a^n \cos n\theta$. Hence find Z-transform of $\cos n\theta$ and $\sin n\theta$. 8,K3,CO5