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Question Paper Code	12456
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B.E./B.Tech - DEGREE EXAMINATIONS, NOV / DEC 2023

Third Semester

Artificial Intelligence and Data Science

(Common to Computer Science and Engineering (AIML) & Computer Science and Engineering (IoT))

20BSMA302 - PROBABILITY AND STATISTICAL MODELLING

(Regulations 2020)

Duration: 3 Hours

Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

- | | <i>Marks,
K-Level, CO</i> |
|--|-------------------------------|
| 1. Define moment generating function. | <i>2,K1,CO1</i> |
| 2. State memory less property for exponential distribution. | <i>2,K1,CO1</i> |
| 3. State central limit theorem. | <i>2,K1,CO2</i> |
| 4. Show that $\frac{2}{5}(2x + 3y), 0 \leq x \leq 1; 0 \leq y \leq 1$ is a joint pdf of X and Y. | <i>2,K2,CO2</i> |
| 5. Define Type-I and Type-II error. | <i>2,K1,CO3</i> |
| 6. State Neyman Pearson lemma. | <i>2,K1,CO3</i> |
| 7. Give the formula to find the rank correlation coefficient in case of tie rank. | <i>2,K1,CO4</i> |
| 8. Give the test statistic used in Kendall test of concordance. | <i>2,K2,CO4</i> |
| 9. Define time-series. | <i>2,K1,CO5</i> |
| 10. Mention the properties of good estimator. | <i>2,K1,CO5</i> |

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) (i) Find the moment generating function of Uniform distribution and hence find its mean and variance. *8,K2,CO1*

(ii) A random variable X has the following probability distribution. *8,K2,CO1*

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find:

(a) The value of K .

(b) $P(1.5 < X < 4.5 / X > 2)$ and

(c) The smallest value of n for which $P(X \leq n) > \frac{1}{2}$.

OR

- b) (i) The contents of urns I, II and III are as follows: 1 white, 2 black and 3 red balls. 2 white, 1 black and 1 red ball, and 4 white, 5 black and 3 red balls respectively. One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urns I? 8,K2,CO1
- (ii) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. what is the probability that he will finally pass the test. 8,K2,CO1
- (a) On the fourth trial and
(b) In less than 4 trials.

12. a) The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal and conditional probability distributions. Also, find $P(X + Y)$ and $P(X + Y > 3)$. 16,K3,CO2

OR

- b) (i) Let X_1, X_2, \dots, X_n be independent identically distributed random variables with mean $\mu = 2$ and variance $\sigma^2 = \frac{1}{4}$. Find $P(192 < S_n < 210)$ where $S_n = X_1 + X_2 + \dots + X_{100}$. 8,K3,CO2
- (ii) Two random variables X and Y have joint density function of X and Y is 8,K3,CO2
- $$f(x, y) = \begin{cases} 2 - x - y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$
- Find Cov (X, Y)

13. a) (i) Ten students got the following percentage of marks in Economics and Statistics. Calculate the coefficient of correlation. 8,K2,CO3

Marks in Economics	78	36	98	25	75	82	90	62	65	39
Marks in Statistics	84	51	91	60	68	62	86	58	53	47

- (ii) The lines of regression of a bivariate population are: 8,K3,CO3
- $$8x - 10y + 66 = 0 ; 40x - 18y = 214.$$
- The variance of x is 9.
Find:
- (a) The mean values of x and y
(b) Correlation coefficient between x and y
(c) Standard deviation of y

OR

- b) Four farmers each used four types of manures for a crop and obtained the yields(in quintals) as below: 16,K2,CO3

		Treatments			
		1	2	3	4
Farmers	A	22	16	21	12
	B	23	17	19	13
	C	21	14	18	11
	D	22	15	19	10

Is there any significant difference between (i) farmers (ii) manures?

14. a) (i) Two methods of instruction to apprentices is to be evaluated. A director assigns 15 randomly selected trainees to each of the two Methods. Due to drop outs, 14 complete in Batch 1 and 12 complete In Batch 2. An achievement test was given to these successful Candidates. Their scores are as follows. 8,K3,CO4

Method I : 70, 90, 82, 64, 86, 77, 84, 79, 82, 89, 73, 81, 83, 66

Method II : 86, 78, 90, 82, 65, 87, 80, 88, 95, 85, 76, 94

Test whether the two methods have significant difference in effectiveness. Use Mann Whitney test at 5% significance level.

- (ii) The clothing manufacture purchased some newly designed sewing Machine in the hopes that production would be increased. The production records of the random sample of the workers are shown below. 8,K3,CO4

Workers 1 2 3 4 5 6 7 8 9 10

Old machine 28 36 27 25 38 36 40 29 32 28

New machine 36 40 25 32 30 32 40 28 35 23

Workers 11 12 13 14 15

Old machine 20 32 32 32 36

New machine 26 31 23 34 36

Use the Wilcoxon – signed rank test to determine whether the new Machines have significantly increased production. Use a 0.05 level of Significance

OR

- b) (i) Use the sign test on the data given below to determine whether there is a statistical increase in the values produced by treatment B over those produced by treatment A: 8,K3,CO4

Subject : 1 2 3 4 5 6 7 8 9 10

Treatment A : 46 41 37 32 28 43 42 51 28 27

Treatment B : 52 43 37 32 31 39 44 53 26 31

- (ii) In 30 tosses of a coin, the following sequence of head (H) and tails (T) is obtained 8,K3,CO4

HTTHTHHHTHHTTHTHTHHTHTTHTHHTHT

(a) Determined the number of runs

(b) Test at 10 % level of significance, whether the sequence is random.

15. a) (i) Find the Maximum Likelihood estimator for θ if 8,K3,CO5
$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x > 0, \theta > 0$$
- (ii) Explain in Detail About ARIMMA Model. 8,K3,CO5
- OR**
- b) Find the ψ weights and π weights for the ARMA (1,1) process given 16,K3,CO5
by $X_t = 0.5 X_{t-1} + Z_t - 0.3Z_{t-1}$.