Reg. No.															

Question Paper Code	13168

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2024

Third Semester

MECHANICAL ENGINEERING

(Common to Mechanical and Automation Engineering)

20BSMA303 - PARTIAL DIFFERENTIAL EQUATIONS AND PROBABILITY THEORY

Regulations - 2020

(Use of statistical table is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (20 × 1 = 20 Marks)

Answer ALL Questions

		Marks	K – Level	CO
1.	If the particular integral of a partial differential equation $\left(D^2 + 2DD' + D'^2\right) z = \cos(x + 2y)$ is $K\cos(x + 2y)$, then $K =$ ______________ . (a) -1/9 (b) 1/9 (c) 1/8 (d) -1/8	1	K2	CO1
2.	The partial differential equations of $z = ax^2 + by^2$ eliminating a and b is (a) $z = xp + yq$ (b) $2z = xp + yq$ (c) $z = xp - yq$ (d) $2z = xp - yq$.	1	K2	CO1
3.	The solution of partial differential equation $\left(D^3 - 2D^2D' - DD'^2 - 2D'^3\right) z = 0$ is (a) $z = f_1(y + x) + f_2(y - x) + f_3(y + 2x)$ (b) $z = f_1(y - 3x) + f_2(y - x) + f_3(y + 2x)$ (c) $z = f_1(y + 9x) + f_2(y - x) + f_3(y + 2x)$ (d) $z = f_1(y + x) + f_2(y - x) + f_3(y - 2x)$	1	K2	CO1
4.	The partial differential equation of $z = ax + by + ab$ by eliminating a and b is (a) $z = px + qy + pq$ (b) $z = px - qy + pq$ (c) $z = px + qy - pq$ (d) $z = -px - qy - pq$	1	K2	CO1
5.	If $B^2 - 4AC < 0$,then $A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + f\left[x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right] = 0$ is classified as (a) Elliptic equation (b) Parabolic equation (c) Hyperbolic equation (d) Circular equation	1	K1	CO2
6.	How many boundary conditions are required to solve the one dimensional wave equations? (a) Four (b) Two (c) Three (d) One	1	K1	CO2
7.	The suitable solution of one dimensional heat flow equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ is (a) $u(x,t) = (A\cos Px + B\sin Px)(C\sin pat + D\cos pat)$ (b) $u(x,t) = (A\cos Px - B\sin Px)(C\sin pat - D\cos pat)$ (c) $u(x,t) = (A\cos Px + B\sin Px)e^{-\alpha^2 P^2 t}$ (d) None of the above	1	K1	CO2
8.	In the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ what does c^2 stand for? (a) $\frac{T}{m}$ (b) $\frac{m}{T}$ (c) $\frac{T^2}{m}$ (d) $\frac{T}{m^2}$	1	K1	CO2
9.	The Fourier cosine transform of e^{-3x} is_______ (a) $\sqrt{\frac{2}{\pi}}\left[\frac{s}{s^2+3^2}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{3}{s^2+3^2}\right]$ (c) $\sqrt{\frac{1}{\pi}}\left[\frac{s}{s^2+3^2}\right]$ (d) None of the these	1	K2	CO3

10. The linear property on Fourier transform states that $F[af(x)+bg(x)]=$ *1 K1 CO3*

(a) $aG(s)+bF(s)$ (b) $aF(s)-bG(s)$ (c) $aF(s)-bG(s)$ (d) $aF(s)+bG(s)$

11. The Fourier sine transform of e^{-ax}, a>0 is_____ *1 K2 CO3*

(a) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+a^2}\right)$ (b) $\sqrt{\frac{2}{\pi}}\left(\frac{a}{s^2-a^2}\right)$ (c) $\sqrt{\frac{2}{\pi}}\left(\frac{a}{s^2+a^2}\right)$ (d) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2-a^2}\right)$

12. The Fourier transform of the convolution of two functions $f(x)$ and $g(x), F(f(x)*g(x))$ is *1 K1 CO3*

(a) $F(f(x))*F(g(x))$ (b) $f(x)g(x)$ (c) $F(f(x))F(g(x))$ (d) $f(x)*g(x)$

13. A random variable whose set of possible values is either finite or countably infinite is called *1 K1 CO4*

(a) continuous random variable
(b) discrete random variable
(c) random process
(d) None of these

14. The c.d.f of a continuous random variable is given by $F(x)=\begin{cases}0 & , \quad x<0\\ 1-e^{-x/5} & , \quad 0\le x<\infty\end{cases}$, then the probability density function is *1 K2 CO4*

(a) $\frac{1}{3}e^{-x/5}$ (b) $\frac{1}{2}e^{-x/5}$ (c) $\frac{1}{5}e^{-x/5}$ (d) $\frac{1}{15}e^{-x/5}$

15. If the pdf of a random variable X is given by $f(x)=\begin{cases}\frac{1}{4}, & -2<x<2\\ 0, & \text{elsewhere}\end{cases}$ then find the mean value of X. *1 K2 CO4*

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

16. In Binomial distribution, $E(X)$is *1 K1 CO4*

(a) np (b) npq (c) n^2p^2 (d) $np-npq$

17. Let (X,Y) be a two independent continuous random variable with joint pdf $f(x,y)$then *1 K1 CO5*

(a) $f(x,y)=f_Y(y)$
(b) $f(x,y)=f_X(x)$
(c) $f(x,y)=f_X(x)f_Y(y)$
(d) $f(x,y)\neq f_X(x)f_Y(y)$

18. If $f(x,y)=k(1-x-y),\ 0<x,y<\frac{1}{2}$ is a joint density function, then the value of k is *1 K3 CO5*

(a) $\frac{1}{2}$ (b) 4 (c) 8 (d) $\frac{1}{8}$

19. The rank correlation coefficient lies between *1 K1 CO5*

(a) 0 and 1 (b) 1 and -1 (c) -1 and 1 (d) None of these

20. If X and Y are two independent random variables then $E[XY]=E(X)-E(Y)$ which implies $Cov(X,Y)$ is equal to *1 K1 CO5*

(a) 1 (b) 0 (c) -1 (d) None of these

PART - B (10 × 2 = 20 Marks)

Answer ALL Questions

21. Find the PDE by eliminating the arbitrary constants a & b from$z=(x^2+a)(y^2+b)$. *2 K2 CO1*
22. Solve $(D-1)(D-D'+1)\,z=0$. *2 K2 CO1*
23. Classify the equation $f_{xx}+2f_{xy}+4f_{yy}=0$. *2 K2 CO2*
24. A rod 30 cm long has it ends A and B kept at$20^o c$and $80^o c$ respectively. Until the steady state condition prevails Illustrate the steady state temperature in the rod. *2 K2 CO2*
25. State Fourier integral theorem. *2 K1 CO3*
26. Find the Fourier cosine transform of $e^{-ax}x\ge 0$. *2 K2 CO3*
27. Given the probability density function$f(x)=\frac{k}{1+x^2}, -\infty<x<\infty$, find the value of k. *2 K2 CO4*
28. State the memoryless property of geometric distribution. *2 K1 CO4*

29. Define covariance of X, Y and coefficient of correlation between X and Y. *2 K1 CO5*

30. Find the correlation co-efficient r_{XY} if cov(x,y) = 12, Var(x) = 9, Var (y) = 25. *2 K2 CO5*

PART - C (6 × 10 = 60 Marks)

Answer ALL Questions

31. a) Form the PDE by eliminating the arbitrary function φ from $\varphi(x^2 + y^2 + z^2, ax + by + cz) = 0$. *10 K3 CO1*

OR

b) Solve: $\left(D^2 - 3DD' + 2D'^2\right) z = 8\sin(x + 3y)$. *10 K3 CO1*

32. a) A string is stretched and fastened to points at a distance 'l' apart .Motion is started by displacing the string in to the form $y = a\sin\left(\frac{\pi x}{l}\right)$, $0 < x < l$, from which it is released at time $t = 0$. Examine the displacement at any time t. *10 K3 CO2*

OR

b) An infinitely long rectangular plate with insulated surface is 10cm wide. The two long edges and one short edges are kept at zero temperature , while the other short edge x = 0 is kept at temperature given by $u(0, y) = \begin{cases} 20y, & 0 \le y \le 5 \\ 20(10 - y), & 5 \le y \le 10 \end{cases}$ .Examine the steady state temperature at any point. *10 K3 CO2*

33. a) Find the Fourier transforms of $f(x) = \begin{cases} 1 - |x| if |x| < 1 \\ 0 for |x| > 1 \end{cases}$ and hence deduce the value of $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. *10 K3 CO3*

OR

b) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier transforms. *10 K3 CO3*

34. a) A random variable *X* has the following probability distribution. *10 K3 CO4*

X	0	1	2	3	4	5	6	7
P(*x*)	0	*K*	2*K*	2*K*	3*K*	K^2	$2K^2$	$7K^2 + K$

Find:

(i) The value of *K*

(ii) $P(1.5 < X < 4.5 / X > 2)$

(iii) Evaluate $P(X<6)$, $P(X \geq 6)$, and $P(0<X<5)$.

OR

b) Assume that the reduction of a person's oxygen consumption during a period of Transcendental Meditation (T.M) is a continuous random variable X normally distributed with mean 37.6 cc/mm and S.D 4.6 cc/min. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by *10 K3 CO4*

(i) At least 44.5 cc/min

(ii) Atmost 35.0cc/min

(iii) Anywhere from 30.0 to 40.0 cc/mm.

35. a) The joint probability mass function of (X, Y) is given by $P(x, y) = k(2x + 3y)$, $x = \{0, 1, 2\}$; $y = \{1, 2, 3\}$. Find all the marginal and conditional probability distributions. *10 K3 CO5*

OR

b) Obtain the equation of the lines of regression from the following data: *10 K3 CO5*

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

36. a) i) Let the p.d.f of X be given by $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}}, & x \geq 0 \\ 0, & otherwise \end{cases}$. *5 K3 CO4*

Compute i) $P\left(X > \frac{1}{2}\right)$ ii) Moment generating function for X.

ii) Let the joint p.d.f .of$(X, Y)$is given by$f(x, y) = \begin{cases} Cxy^2, & 0 \leq x \leq y \leq 1 \\ 0, & otherwise \end{cases}$. *5 K3 CO5*

Determine the

(i) Value of C

(ii) Marginal p.d.f. of X&Y.

OR

b) i) Starting at 5.00 am every half hour there is a flight from san Francisco airport to Los Angeles International Airport. Suppose that none of these planes is completely sold out and that they always have room for passengers. A person who wants to fly to L.A. arrives at the airport at a random time between 8.45 a.m. and 9.45 a.m. find the probability that she waits 1) at most 10 mins. 2) at least 15 mins. *5 K3 CO4*

ii) The joint pdf of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2; 0 \leq y \leq 1$. Compute (1) $P[X > 1]$ (2) $P\left[Y < \frac{1}{2}\right]$. *5 K3 CO5*