Reg. No.																

Question Paper Code	14186

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025

Third Semester

Mechanical Engineering

20BSMA303 - PARTIAL DIFFERENTIAL EQUATIONS AND PROBABILITY THEORY

Regulation - 2020

(Use of Statistical Tables is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (10 × 1 =10 Marks)

Answer ALL Questions

Marks | K – Level | CO

1. Elimination of arbitrary constants a and b from $z = ax + by + ab$ will result in the partial differential equation: *1 K3 CO1*
   (a) $z = px + qy + pq$ (b) $z = px + qy + p^2$
   (c) $z = qx + py + q^2$ (d) $z = qx + py + pq$
2. The general solution of the Lagrange's linear equation $px + qy = z$ is *1 K3 CO1*
   (a) $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ (b) $f\left(xy, \frac{y}{z}\right) = 0$ (c) $f\left(\frac{x}{y}, yz\right) = 0$ (d) $f(xy, yz) = 0$
3. The second order partial differential equation $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ is *1 K2 CO2*
   (a) elliptic (b) parabolic (c) hyperbolic (d) quadratic
4. Which of the following is **not** the solution of one-dimensional heat equation? *1 K1 CO2*
   (a) $u(x,t) = (A cospx + B sinpx)e^{-\alpha^2 p^2 t}$ (b) $u(x,t) = (A_1 cospx + B_1 sinpx)e^{\alpha^2 p^2 t}$
   (c) $u(x,t) = (A_2 e^{px} + B_2 e^{-px})e^{-\alpha^2 p^2 t}$ (d) $u(x) = (A_3 x + B_3)$
5. If $F\{f(x)\} = F(s)$ then $F\{f(ax)\} =$ *1 K1 CO3*
   (a) $-\frac{1}{a}F\left(\frac{s}{a}\right)$ (b) $\frac{1}{a}F\left(\frac{s}{a}\right)$ (c) $\frac{1}{|a|}F\left(\frac{s}{a}\right)$ (d) $aF(as)$
6. Which of the following functions is self-reciprocal under Fourier transform? *1 K1 CO3*
   (a) e^{-x^2} (b) e^{x^2} (c) $e^{-\frac{x^2}{2}}$ (d) $e^{\frac{x^2}{2}}$
7. If $f(x) = kx^2, 0 < x < 3$ is to be a density function, then the value of k is *1 K3 CO4*
   (a) $\frac{2}{9}$ (b) $\frac{1}{9}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$
8. If the MGF of a continuous random variable X is $\frac{e^{5t} - e^{4t}}{t}, t \neq 0$, then $E(X)$ is *1 K2 CO4*
   (a) $\frac{1}{2}$ (b) $\frac{9}{2}$ (c) $\frac{1}{12}$ (d) $\frac{4}{5}$
9. For the following joint probability distribution of (X, Y), the value of k is *1 K3 CO5*

	Y		
X	1	2	3
0	$3k$	$6k$	$9k$
1	$5k$	$8k$	$11k$
2	$7k$	$10k$	$13k$

   (a) $\frac{1}{24}$ (b) $\frac{1}{36}$ (c) $\frac{1}{54}$ (d) $\frac{1}{72}$

10. If $f(x,y) = k(1 - x - y),\ 0 < x, y < \frac{1}{2}$ is a joint density function, then the value of k is *1 K3 CO5*
    (a) $\frac{1}{2}$ (b) 4 (c) 8 (d) $\frac{1}{8}$

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

11. Form the partial differential equation by eliminating the arbitrary constants *a, b* from $z = (x^2 + a)(y^2 + b)$. *2 K3 CO1*

12.	Find the complete integral of $p + q = 1$.	2	K3	CO1
13.	Solve $p = 2qx$.	2	K3	CO1
14.	Classify the partial differential equation $u_{xx} - 2u_{xy} = 0$.	2	K2	CO2
15.	State the assumptions made in deriving one dimensional wave equation.	2	K1	CO2
16.	A rod 10cm long has its ends A and B kept at $30°C$ and $50°C$ respectively until steady state conditions prevail. Find the steady state temperature in the rod.	2	K2	CO2
17.	State the Parseval's identity on Fourier transform.	2	K1	CO3
18.	Write down the Fourier cosine transform pair.	2	K1	CO3
19.	A random variable X has the following probability distribution:	2	K3	CO4

x	-2	-1	0	1	2	3
$p(x)$	0.1	k	0.2	$2k$	0.3	$3k$

Find k.

20.	The mean of a Binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution.	2	K2	CO4
21.	If the joint pdf of (X, Y) is given by $f(x,y) = e^{-(x+y)}, 0 \leq x, y < \infty$, are X and Y independent?	2	K3	CO5
22.	The equations of two regression lines got in a correlation analysis are $3x + 12y = 19$ and $3y + 9x = 46$. Obtain the mean values of X and Y.	2	K2	CO5

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. — 11 K3 CO1

OR

b) Solve$\left[D^3 - 7DD'^2 - 6D'^3\right]z = sin(x + 2y) + e^{2x+y}$. — 11 K3 CO1

24. a) A tightly stretched string with fixed end points$x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 sin^3\left(\frac{\pi x}{l}\right)$. If it is released from the rest from this position, find the displacement y at any time and at any distance from the end $x = 0$. — 11 K3 CO2

OR

b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at the short edge $y = 0$ is given by $u(x, 0) = \begin{cases} 20x, & 0 \leq x \leq 5 \\ 20(10 - x), & 5 \leq x \leq 10 \end{cases}$ and all the other three edges are kept at $0°C$. Find the steady state temperature at any point in the plate. — 11 K3 CO2

25. a) Find the complex Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence deduce that $\int_0^\infty \frac{sint - tcost}{t^3} dt = \frac{\pi}{4}$. — 11 K3 CO3

OR

b) Find the Fourier cosine transform of $f(x) = e^{-ax}, a > 0$ and$g(x) = e^{-bx}, b > 0$, hence evaluate$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$. — 11 K3 CO3

26. a) A discrete random variable X has the probability function: — 11 K3 CO4

x	1	2	3	4	5	6	7	8
$P(x)$	$2a$	$4a$	$6a$	$8a$	$10a$	$12a$	$14a$	$4a$

(i) Find the value of a

(ii) Find $P(X \geq 3), P(X < 3)$

(iii) Find the distribution function.

OR

	b) (i)	Find the moment generating function of Poisson distribution.	5	K3	CO4
	(ii)	The mileage which car owners get with certain kind of radial tyre is a random variable having an exponential distribution with mean 4000 km. Find the probabilities that one of these tyres will last (1) at least 2000 km (2) at most 3000 km.	6	K3	CO4
27.	a)	Find the coefficient of correlation between X and Y from the data given below.	11	K3	CO5

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

OR

	b)	The lifetime of a particular variety of electric bulbs may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using central limit theorem find the probability that the average life time of 60 bulbs exceeds 1250 hours.	11	K3	CO5
28.	a) (i)	Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) At least one boy.	6	K3	CO4
	(ii)	The lines of regression of a bivariate population are: $8x - 10y + 66 = 0$ and $40x - 18y = 214$. Find i) the mean values of X and Y ii) the correlation coefficient between X and Y.	5	K3	CO5

OR

	b) (i)	Let X follows normally distribution with mean of X is 12 and standard deviation is 4. Find $P(X \geq 20)$.	6	K3	CO4
	(ii)	The joint pdf of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2; 0 \leq y \leq 1$. Compute (1) $P[X > 1]$ (2) $P\left[Y < \frac{1}{2}\right]$.	5	K3	CO5