Reg. No.																

Question Paper Code	14696

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2026

Third Semester

Mechanical Engineering

(Common to Mechanical and Automation Engineering)

20BSMA303 - PARTIAL DIFFERENTIAL EQUATIONS AND PROBABILITY THEORY

Regulations - 2020

(Use of Statistical *Table* is permitted)

Duration: 3 Hours | Max. Marks: 100

PART – A (i) (MCQ) (16 × 1 = 16 Marks)

Answer ALL Questions

		Marks	K – Level	CO
1.	The p.d.e by eliminating the arbitrary constants a & b from $z = ax + by$ is a) $z = px + qy$ b) $z = pq$ c) $z = pxqy$ d) $z = pq + 1$	1	K1	CO1
2.	Solution of p.d.e $px + qy = z$ is a) $f\left(\frac{x}{y}, \frac{x}{z}\right) = 0$ b) P p + Q q = R c) $f\left(\frac{x}{y}, \frac{x}{zy}\right) = 0$ d) $f\left(\frac{x}{z}, \frac{x}{zy}\right) = 0$	1	K2	CO1
3.	The Particular Integral of $(D^2 + DD')z = e^{x-y}$ a) $x\, ye^{x-y}$ b) $e^x/2$ c) ye^{x-y} d) xe^{x-y}	1	K3	CO1
4.	The one dimensional heat flow equation in steady state is a) $\frac{\partial u}{\partial t} = 0$ b) $\frac{du}{dt} = 0$ c) $\frac{\partial^2 u}{\partial t^2} = 0$ d) $\frac{d^2u}{dt^2} = 0$	1	K1	CO2
5.	The nature of the PDE $u_{xx} + u_{xy} + u_{yy} = 0$ is a) Hyperbolic b) Parabolic c) Elliptic c) All the above	1	K2	CO2
6.	The ends A and B of a rod of length 20cm long have their temperature kept at 300c and 900c.The steady state temperature distribution on the rod is a) U(x) = 3x + 30 b) U(x) = 5x + 20 c) U(x) = 7x + 20 d) None of the above.	1	K3	CO2
7.	In Fourier transform F(p)=$\int_{-\infty}^{\infty} e^{ipx} f(x)dx$, e^{ipx} is said to be a) Not a Kernel function b) Kernel function c) Periodic function d) Aperiodic function	1	K1	C3
8.	Fourier Transform of $f(x - a)$ is a) $F\big(f(x)\big) = e^{ias}$ F(s) b) $F(f(x)) = e^{-ias}$ F(s) c) $F(f(x)) = e^{ix}$ F(s) d) $F(f(x)) = e^{is}$ F(s)	1	K2	CO3
9.	*The value of* $\int_{-\infty}^{\infty} e^{-x^2} dx$ is a) $\frac{\sqrt{\pi}}{3}$ b) $\frac{\sqrt{\pi}}{2}$ c) $\frac{\pi}{2}$ d) $\sqrt{\pi}$	1	K3	CO3
10.	The expected value of a discrete random variable 'X' is given by _____ a) P(X) b) $\sum$ x P(X) c) $\sum$ P(X) d) 1	1	K1	CO4
11.	If $P(1) = P(3)$ in Poisson's distribution, what is the mean? a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{6}$ d) $\sqrt{7}$	1	K2	CO4

12. If the random variable X has the following p.d.f $f(x) = K(1+x),\ 2 \le x \le 5$. the value of K is — *1 K3 CO4*

a) 5 b) 8
c) 6 d) 2

13. If X and Y are two independent random variables then — *1 K1 CO5*

a) $E(XY) = 1$ b) $E(XY) = 0$
c) $E(XY) = E(X).E(Y)$ d) $E(XY) =$ a constant

14. Consider the following table — *1 K3 CO5*

X / Y	0	1
-1	1/8	2/8
1	3/8	2/8

Then the marginal probability distribution at X = 0 is

a) 4/8 b) 2/8
c) -3/8 d) 5/8

15. The two lines of regression are $4x - 5y + 33 = 0, 20x - 9y = 107$. The mean of X and Y are — *1 K2 CO5*

a) (12,17) b) (17,13)
c) (13,17) d) (17,12)

16. If $X_1, X_2, \dots, X_n$ are Poisson variates with parameter $\lambda = 2$, and $S_n = X_1 + X_2 + \cdots + X_n$ then $S_n$follows — *1 K3 CO5*

a) N(150,$\sqrt{150}$) b) N(150, 150)
c) N(150,$\sqrt{160}$) d) N(150,160)

PART – A (ii) (Short Answer Questions) (8 × 3 = 24 Marks)

Answer ALL Questions

17. Find the general solution of $pzx + qzy = xy$. — *3 K3 CO1*
18. Find the particular integral of $(D^2 - 7DD' + 6D'^2)z = x + y$. — *3 K4 CO1*
19. Classify the PDE $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8u_y = 0$. — *3 K3 CO2*
20. The ends A and B of a rod of length 20 cm long have their temperature kept 30^0 C and 80^0 C until steady state prevails. Find the steady state temperature on the rod. — *3 K4 CO2*
21. Find the Fourier cosine Transform of $e^{-2x} + 2e^{-x}$. — *3 Kx CO3*
22. If the random variable X has the following probability distribution — *3 K3 CO4*

X : -2 -1 0 1 2 3
P(X) : 0.1 k 0.2 2k 0.3 3k Find the value of k.

23. For the binomial distribution n = 6, p = 0.4, q = 0.6. Find the mean. — *3 K3 CO4*
24. The Regression equations of (X,Y) are $3x + 2y = 26, 6x + y = 31$, find the regression coefficient of Y on X. — *3 K4 CO5*

PART - B (Descriptive Answer Questions)

(5 × 12 = 60 Marks)

Answer ALL Questions

25. a) Form the partial differential equation by eliminating arbitrary function Φ from $\Phi(x^2 + y^2 + z^2, ax + by + cz) = 0$. — *12 K3 CO1*

OR

b) Solve $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$. — *12 K3 CO1*

26. a) A string is stretched and fastened to two points that are distinct string l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$from which it is released at time t=0. Find the displacement of any point on the string at a distance of x from one end at time t. — *12 K4 CO2*

OR

b) An infinitely long rectangular plate with insulated surfaces is 20 cm wide. The two long edges and one short edge are kept at 0^0C, while the other short edge x=0 is kept at temperature $= \begin{cases} 10y & 0 \le y \le 10 \\ 10(20-y), & 10 \le y \le 20 \end{cases}$. Find the steady state temperature distribution in the plate. *12 K4 CO2*

27. a) Find the Fourier Transform of $f(x) = \begin{cases} 1, |x| \le a \\ 0, |x| > a > 0 \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right) dt$. *12 K3 CO3*

OR

b) Using Parseval's Identity evaluate the following integrals. $(i) \int_0^\infty \frac{dx}{(a^2+x^2)^2}, (ii) \int_0^\infty \frac{x^2 dx}{(a^2+x^2)^2} \quad a > 0.$ *12 K3 CO3*

28. a) The probability mass function of a discrete R. V X is given in the following table Find (i) the value of a , (ii) $P(X < 3)$, (iii) Mean of X, (iv) Variance of X. *12 K3 CO4*

X	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

OR

b) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and Standard Deviation of 60 hours. Find the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours and (iii) between 1920 hours and 2160 hours. *12 K3 CO4*

29. a) From the following table for bivariate distribution of (X, Y). Find *12 K4 CO5*
(i) $P(X \le 1)$ (ii)$P(Y \le 3)$ (iii) $P(X \le 1, Y \le 3)$
(iv) $P(X \le 1/\ Y \le 3)$ (v) $P(Y \le 3/\ X \le 1)$ (vi) $P(X + Y \le 4)$

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

OR

b) If $f(x,y) = \frac{6-x-y}{8}$, $0 \le x \le 2$, , $2 \le y \le 4$ for a bivariate random variable (X,Y), Find the correlation coefficient ρ. *12 K4 CO5*