Reg. No.																

Question Paper Code	12904

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2024

Third Semester

Mechanical Engineering

(Common to Mechanical and Automation Engineering)

20BSMA303 - PARTIAL DIFFERENTIAL EQUATIONS AND PROBABILITY THEORY

Regulations - 2020

(Use of Statistical Table is permitted)

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
1.	Form a partial differential equation by eliminating the arbitrary constants from $z = ax^2 + by^2$.	2	K2	CO1
2.	Find the complete integral of $p + q = 1$.	2	K2	CO1
3.	Classify the PDE $f_{xx} - 2f_{xy} = 0 \; for \; x > 0 \; \& \; y > 0$.	2	K1	CO2
4.	A rod 30cm long has its ends A and B kept at $20°C$ and $80°C$ respectively until steady state conditions prevail. Find the steady state temperature in the rod.	2	K2	CO2
5.	State change of scale property of Fourier Transform.	2	K1	CO3
6.	Find sine transform of $\frac{1}{x}$.	2	K1	CO3
7.	A discrete R.V X has moment generating function $M_X(t) = \left(\frac{1}{4} + \frac{3}{4}e^t\right)^5$ Find $E(X)$.	2	K2	CO4
8.	State the memory less property of Geometric distribution.	2	K1	CO4
9.	A continuous random variable X follows the probability law $f(x) = Ax^2, 0 \le x \le 1$. Determine A.	2	K2	CO5
10.	Can $y = 5 + 2.8x$ and $x = 3 - 0.5y$ be the regression lines of Y on X and X on Y respectively. Give suitable arguments.	2	K2	CO5

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
11. a) i)	Solve $(mz - ny)p + (nx - lz)q = ly - mx$.	8	K3	CO1
ii)	Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2 + z^2, x + y + z) = 0$.	8	K3	CO1

OR

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create

b) i) Solve $\left[D^3 - 7DD'^2 - 6D'^3\right]z = \sin(x + 2y) + e^{2x+y}$. *8 K3 CO1*

ii) Solve $z = px + qy + p^2q^2$. *8 K3 CO1*

12. a) A string is stretched and fastened to two end points l apart. Motion is started by displacing the string in to the form $y = k(lx - x^2)$ from which it is released at a time $t = 0$. Find the displacement of any point of the string at a distance x from one end at any time t. *16 K3 CO2*

OR

b) A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. the temperature at shot edge $y = 0$ is given by $u(x) = \begin{cases} 20x, & 0 \le x \le 5 \\ 20(10 - x), & 5 \le x \le 10 \end{cases}$ and all other three edges are kept at$0°C$. Find the steady state temperature at any point in the plate. *16 K3 CO2*

13. a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \ge 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{sin^2 t}{t^2} dt$ and $\int_0^\infty \frac{sin^4 t}{t^4} dt$. *16 K3 CO3*

OR

b) i) Prove that $f(x) = e^{-x^2/2}$ is self-reciprocal under the Fourier cosine transform. *8 K3 CO3*

ii) Evaluate $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$. *8 K3 CO3*

14. a) i) A discrete random variable *X* has the probability function, *8 K3 CO4*

x	1	2	3	4	5	6	7	8
p(x)	*2a*	*4a*	*6a*	*8a*	*10a*	*12a*	*14a*	*4a*

1. Find the value of *a*
2. Find $P(X \ge 3), P(X < 3)$
3. Find the distribution function.

ii) Let X follows normally distribution with mean of 12 and Standard deviation is 4. Find the probability of the following: *8 K3 CO4*

(i) $X \ge 20$ (ii) $0 \le X \le 12$.

OR

b) i) The mileage which car owners get with certain kind of radial tyre is a random variable having an exponential distribution with mean 4000 km. Find the probabilities that one of these tyres will last (1) atleast 2000 km (2) at most 3000km. *8 K3 CO4*

ii) In a bolt factory, machines A, B, C manufacture 25%, 35%, 40% of the total output respectively. Out of their outputs 5, 4, 2 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A? *8 K3 CO4*

15\. a) The joint probability mass function of (X, Y) is given by $P(x, y) = k(2x + 3y)$, $x = 0, 1, 2, y = 1, 2, 3$. Find the marginal probability distributions. Also find the probability distribution of $X + Y$ and the conditional probability $P(X/Y = 1)$. *16 K3 CO5*

OR

b) i) Find the coefficient of correlation between X and Y from the data given below. *10 K3 CO5*

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

ii) Let $X_1, X_2, \ldots\ldots, X_n$ be Poisson variates with parameter $\lambda = 2$. Let $S_n = X_1 + X_2 + \ldots + X_n$, where $n = 75$, Find $P[120 \leq S_n \leq 160]$ using the central limit theorem. *6 K3 CO5*