Reg. No.												

Question Paper Code	12455

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2023

Third Semester

MECHANICAL ENGINEERING

(Common to Mechanical and Automation Engineering)

20BSMA303 – PARTIAL DIFFERENTIAL EQUATIONS AND PROBABILITY THEORY

(Regulations 2020)

(Use of Statistical Table is permitted)

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

		Marks, K-Level, CO
1.	Form the PDE by eliminating the arbitrary constants a, b from $z = (x^2 + a)(y^2 + b)$.	2,K2,CO1
2.	Find the particular integral of $(D^2 - 2DD' + D'^2)z = e^{x-y}$.	2,K2,CO1
3.	Classify the PDE $u_{xx} - 2u_{xy} + u_{yy} = 0$.	2,K1,CO2
4.	A rod 30cm long has its ends A and B kept at $20°C$ and $80°C$ respectively until steady state conditions prevail. Find the steady state temperature in the rod.	2,K1,CO2
5.	State change of scale property of Fourier Transform.	2,K1,CO3
6.	Write Fourier sine transform pair.	2,K1,CO3
7.	Find the mean and variance of the distribution whose moment generating function is $(0.6 + 0.4e^t)^4$.	2,K2,CO4
8.	State memory less property of geometric distribution.	2,K1,CO4
9.	The joint p.d.f of the random variable (X, Y) is $f(x,y) = \begin{cases} cxy & , 0 < x < 2 \; ; 0 < y < 2 \\ 0 & , \quad otherwise \end{cases}$. Find the value of *c*.	2,K2,CO5
10.	Can $y = 5 + 2.8x$ and $x = 3 - 0.5y$ be the regression lines of *Y* on *X* and *X* on *Y* respectively. Give suitable arguments.	2,K2,CO5

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) (i) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. *8,K3,CO1*

(ii) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2 + z^2, x + y + z) = 0$. *8,K3,CO1*

OR

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create ***12455***

b) (i) Solve $\left[D^3 - 7DD'^2 - 6D'^3\right]z = \sin(x+2y) + e^{2x+y}$. *8,K3,CO1*

(ii) Solve $z = px + qy + p^2q^2$ *8,K3,CO1*

12. a) A string is stretched and fastened to two end points l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at a time $t = 0$. Find the displacement of any point of the string at a distance x from one end at any time t. *16,K3,CO2*

OR

b) A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at shot edge $y = 0$ is given by $u(x) = \begin{cases} 20x, & 0 \le x \le 5 \\ 20(10-x), & 5 \le x \le 10 \end{cases}$ and all other three edges are kept at $0°C$. Find the steady state temperature at any point in the plate. *16,K3,CO2*

13. a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| \ge 1 \end{cases}$. Hence deduce that $\int_0^\infty \frac{sint - tcost}{t^3} dt = \frac{\pi}{4}$ and $\int_0^\infty \left(\frac{sint - tcost}{t^3}\right)^2 dt = \frac{\pi}{15}$. *16,K3,CO3*

OR

b) (i) Prove that $f(x) = e^{-x^2/2}$ is self-reciprocal under the Fourier cosine transform. *8,K3,CO3*

(ii) Evaluate $\int_0^\infty \frac{x^2}{(x^2+16)(x^2+9)} dx$. *8,K3,CO3*

14. a) (i) A discrete random variable *X* has the probability function, *8,K3,CO4*

x	1	2	3	4	5	6	7	8
p(x)	*2a*	*4a*	*6a*	*8a*	*10a*	*12a*	*14a*	*4a*

1. Find the value of *a*.
2. Find $P(X \ge 3), P(X < 3)$.
3. Find the distribution function.

(ii) A certain type of storage battery lasts on the average 3.0 years with standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years. *8,K3,CO4*

OR

b) (i) In a company the monthly break down of a machine is a random variable with Poisson distribution, with an average 1.8. Find the probability that the machine will function for a month (a) Without break down, (b) With exactly one break down, (c) With at least one break down. *8,K3,CO4*

(ii) The contents of urns I, II and III are as follows: 1 white, 2 black and 3 red balls. 2 white, 1 black and 1 red ball, and 4 white, 5 black and 3 red balls respectively. One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urn I. *8,K3,CO4*

15. a) The joint probability mass function of (X, Y) is given by $P(x, y) = k(2x + 3y)$, $x = 0, 1, 2, y = 1, 2, 3$. Find the marginal probability distributions. Also find the conditional probability $P(X/Y = 1)$. *16,K3,CO5*

OR

b) (i) Find the coefficient of correlation between X and Y from the data given below. *8,K3,CO5*

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

(ii) The lifetime of a particular variety of electric bulbs may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using central limit theorem, find the probability that the average life time of 60 bulbs exceeds 1250 hours. *8,K3,CO5*

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