Reg. No.																

Question Paper Code	13169

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2024

Third Semester

Computer Science and Engineering

(Common to Information Technology and M.Tech CSE (5 yrs Integrated))

20BSMA304 - Statistics and Linear Algebra

Regulation - 2020

(Use of statistical table is permitted)

Duration: 3 Hours Max. Marks: 100

PART - A (MCQ) (20 × 1 = 20 Marks)

Answer ALL Questions

Marks K – Level CO

1. The intelligence quotients (IQ's) of 10 boys in a class are given below: 70, 120, 110, 101, 88, 83, 95, 98, 107, and 100. The mean I.Q is *1 K2 CO1*
   (a) 97.2 (b) 97 (c) 97.6 (d) 96.65
2. The first four central moments of a distribution are 0, 2.5, 0.7 and 18.75. The Skewness of the distribution is *1 K2 CO1*
   (a) 0135 (b) 0.001 (c) 0.031 (d) 0.0015
3. A box contains 8 Red, 3 White and 9 Blue balls. If 3 balls are drawn at random, then the probability that all the 3 are red and 1 of each color is drawn is *1 K2 CO1*
   (a) 97.2 (b) 97 (c) 97.6 (d) 96.65
4. On an average typist makes 2 mistakes per page, the probability that she will make exactly 3 or more errors on a page is *1 K2 CO1*
   (a) 0.3278 (b) 0.1514 (c) 0.3233 (d) 0.3333
5. Which test statistic is used to test the hypothesis $H_0 : \sigma^2 = {\sigma_0}^2, H_1 : \sigma^2 \neq {\sigma_0}^2$? *1 K2 CO2*
   (a) F-test (b) t-test (c) z-test (d) χ^2 test
6. The standard error of the difference between the two proportions if $\overline{p}_1 = 0.10, \overline{p}_2 = 0.133, n_1 = 50$ and $n_2 = 75$ is *1 K2 CO2*
   (a) 0.0547 (b) 0.0258 (c) 0.0014 (d) 0.2345
7. A random sample of 625 students from a normal population of unknown mean has mean 10 and S.D 1.5**.** Test statistic value is *1 K2 CO2*
   (a) 16.98 (b) 14.5 (c) 16.6 (d) 15.43
8. What test will be used to test the variances between two samples? *1 K1 CO2*
   (a) t-test (b) χ^2 test (c) F-test (d) z-test
9. Which of the following set is the span of R^3. *1 K1 CO3*
   (a) {(0,0,0), (1,1,1), (1,0,0)} (b) {(1,0,0), (1,1,1), (0,1,0)}
   (c) {(1,0,0), (1,1,1), (0,0,1)} (d) {(1,0,0), (0,1,0), (0,0,1)}
10. Let $M_{m\times n}(F)$ be the vector space of m x n matrices over a field F, then the dimension of $M_{m\times n}(F)$ is *1 K1 CO3*
    (a) n (b) m (c) mn (d) m-n
11. R^n has __________ dimensions. *1 K1 CO3*
    (a) m (b) n (c) 2 (d) 1
12. The dimension of a vector space V is defined as: *1 K1 CO3*
    (a) The number of vectors in V (b) The number of elements in a basis for V
    (c) The total number of subspaces of V (d) The magnitude of the largest vector in V

13. Let V and W be vector spaces and $T:V \rightarrow W$ be linear. If V is finite dimensional, then — 1 K1 CO4

(a) Nullity(T) + rank(T) = dim(V) (b) Nullity(T) - rank(T) = dim(V)

(c) Nullity(T) = rank(T) (d) rank(T) = dim(V)

14. Let $T: R^3 \rightarrow R^2$ be defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. The matrix of transformation with respect to the standard bases of R^3 and R^2 is — 1 K2 CO4

(a) $[T]_B^r = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $[T]_B^r = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $[T]_B^r = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $[T]_B^r = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

15. Let β and γ be the standard ordered bases for R^2 and R^3 respectively. Let $T:R^2 \rightarrow R^3$ defined by $T(a_1, a_2) = (2a_1 - a_2, 3a_1 + 4a_2, a_1)$, the value of $[T]_\beta^\gamma$ is — 1 K2 CO4

(a) $[T]_\beta = \begin{bmatrix} 2 & -1 \\ 1 & 4 \\ 1 & 0 \end{bmatrix}$ (b) $[T]_\beta = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}$

(c) $[T]_\beta = \begin{bmatrix} 1 & -1 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}$ (d) $[T]_\beta = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 1 & 1 \end{bmatrix}$

16. The trace and determinant of a 2 ×2 matrix are known to be – 2 and – 35 respectively then the eigen values of the matrix is — 1 K2 CO4

(a) -7, 5 (b) -7, -5 (c) 7, -5 (d) 7, 5

17. Cauchy-Schwarz Inequality is — 1 K1 CO5

(a) $|\langle x, y\rangle| > \|x\|.\|y\|$ (b) $|\langle x, y\rangle| \le \|x\|.\|y\|$

(c) $|\langle x, y\rangle| \ne \|x\|.\|y\|$ (d) $|\langle x, y\rangle| \le \|x\| + \|y\|$

18. If $x = (5+i, 4), and\ y = (8+i, 7+i)$ then the value of $\|x+y\|$ is — 1 K2 CO5

(a) $\sqrt{295}$ (b) $\sqrt{259}$ (c) $\sqrt{592}$ (d) $\sqrt{285}$

19. Let V be IPS over field F, Then for any $u, v \in V$ and $\alpha, \beta \in F$, if $\|cu\| =$ ___________. — 1 K1 CO5

(a) $\|u\|$ (b) $c\ \|u\|$ (c) $|c|\|u\|$ (d) $\langle u, v\rangle = 0$

20. Linear polynomial in square approximation is — 1 K1 CO5

(a) $y = ct^3 + dt + e$ (b) $y = ct^2$ (c) $y = ct^2 + e$ (d) $y = dt + e$

PART - B (10 × 2 = 20 Marks)

Answer ALL Questions

21. Mention various types of skewness. — 2 K1 CO1

22. Define Poisson distribution. — 2 K1 CO1

23. Define Type I and Type II errors. — 2 K2 CO2

24. Given that n=22, sample mean=153.7, population mean=146.3 and S.D=17.2. Find the test statistics value. — 2 K2 CO2

25. State and prove that cancellation law for vector addition. — 2 K2 CO3

26. For R^3, determine whether the first vector can be expressed as a linear combination of the other two vectors $(-2,0,3), (1,3,0), (2,4,-1)$ — 2 K2 CO3

27. Suppose that $T: R^2 \rightarrow R^2$ is linear, T(1,0) = (1,4), and T(l, 1) = (2,5).What is T(2,3)? — 2 K2 CO4

28. Compute the matrix of the transformation with respect to the standard basis $P_2(R)$. Let $T: P_2(R) \rightarrow P_2(R)$ defined as $T(f(x)) = f(x) + xf'(x)$ — 2 K3 CO4

29. State triangle inequality. — 2 K1 CO5

30. What are advantages of least square approximation? *2 K1 CO5*

PART - C (6 × 10 = 60 Marks)

Answer ALL Questions

31. a) An experiment consists of throwing two six-sided dice and observing the number of spots on the upper faces. Determine the probability (i) The sum of the spots is 3. (ii) Each die shows four or more spots. (iii) The sum of the spots is not 3. (iv) Neither a one nor a six appear on each die. (v) A pair of sixes appear. (vi) The sum of the spots is 7. *10 K3 CO1*

OR

b) From the following data, find (i) The two regression equations (ii) The coefficient of correlation between the marks in mathematics and statistics (iii) The most likely marks in statistics when marks in mathematics are 30 *10 K3 CO1*

Marks in Math's	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

32. a) Two random samples give the following results. *10 K3 CO2*

Sample	Size	Sample mean	Sum of squares of deviation from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population.

OR

b) From a poll of 800 television viewers, the following data have been accumulated as to, their levels of education and their preference of television stations. We are interested in determining if the selection of a TV station is independent of the level of education. *10 K3 CO2*

	Educational Level			
	High School	Bachelor	Graduate	Total
Public Broadcasting	50	150	80	280
Commercial Stations	150	250	120	520
Total	200	400	200	800

(i) State the null and the alternative hypotheses.
(ii) Show the contingency table of the expected frequencies.
(iii) Compute the test statistic.
(iv) The null hypothesis is to be tested at 95% confidence. Determine the Critical value for this test.

33. a) Show that $F^n = \{(a_1, a_2, a_3, \dots a_n) : a_i \in F\}$ is a vector space over F with respect to addition and scalar multiplication defined component wise. *10 K3 CO3*

OR

b) i) Determine whether $\{x^2 + 3x - 2, 2x^2 + 5x - 3, -x^2 - 4x - 4\}$ forms a basis for $P_2(R)$. *5 K3 CO3*

ii) Determine 'x' so that $(1, -1, x-1), (2, x, -4), (0, x+2, -8)$ in R^3 are linearly dependent over R. *5 K3 CO3*

34.	a)	Let T: $P_2(R) \rightarrow P_3(R)$ defined by $T(f(x)) = f'(x) + \int_0^x 2f(t)dt$ Find the basis for N (T) and R (T) and hence verify the dimension theorem. Is T one-to-one? Is T onto? Justify your answer.	10	K3	CO4
		OR			
	b)	For the linear operator T: $P_2(R) \rightarrow P_2(R)$ defined as $T(f(x)) = f(x) + (x+1)f'(x)$. Find the Eigen values of T and an ordered basis B for $P_2(R)$ such that matrix of the given transformation with respect to the new resultant basis B is a diagonal matrix.	10	K3	CO4
35.	a)	Let $u = (2, 1+i, i)$, $v = (2-i, 2, 1+2i)$ be vectors in $C^3(C)$. Compute using the standard inner product $< u, v >$, $\|u\|$, $\|v\|$ and $\|u+v\|$.	10	K3	CO5
		OR			
	b)	Check that X= (1,1,1), Y=(-1,0,-1), Z=(-1,2,3) in R^3 linearly independent. Using gram-schmidt orthogonalization process computes an Orthogonal basis of R^3 .	10	K3	CO5
36.	a) i)	Let $T: R^2 \rightarrow R^3$ is a linear transformation such that $T(1,1) = (1,0,2), T(2,3) = (1,-1,4)$. Then find (a) T (b) T(2,5), T(8,11) (c) Rank T (d) Is T one-one or onto?	5	K3	CO4
	ii)	Prove that $R^2(R)$ is an inner product space defined for $u = (a_1, a_2)$ and $v = (b_1, b_2)$ by $< u, v > = a_1b_1 - a_2b_1 - a_1b_2 + 2a_2b_2$.	5	K3	CO5
		OR			
	b) i)	Suppose $T: R^2 \rightarrow R^2$ is linear , $T(1,0) = (1,4), T(1,1) = (2,5)$. What is (2,3) ? Is T one- to- one?	5	K3	CO4
	ii)	If u and v are any two vectors in an inner product space, then prove that $\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2)$.	5	K3	CO5