Reg. No.												

Question Paper Code	12413

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2023

Fourth Semester

Electronics and Communication Engineering

(Common to Computer and Communication Engineering)

20BSMA401 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

(Statistical Table may be provided)

(Regulations 2020)

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

Marks, K-Level, CO

1. If X and Y are two independent random variables with variances 2 and 3 respectively. Find the variance of $3X + 4Y$. *2,K1,CO1*
2. Check whether $f(x) = \frac{1}{1+x^2}, -\infty < x < \infty$ is a probability density function or not. *2,K2,CO1*
3. The joint pdf of the random variable (X, Y) is $f(x,y) = \begin{cases} k(x+y), 0 < x < 2; 0 < y < 2 \\ 0, \ otherwise \end{cases}$. Find the value of k. *2,K1,CO2*
4. Define Regression. *2,K1,CO2*
5. Define a wide sense stationary process. *2,K1,CO3*
6. Write any two properties of autocorrelation. *2,K1,CO3*
7. State any two properties of a Poisson process. *2,K1,CO4*
8. Let A=$\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ be a stochastic matrix. Check whether it is regular. *2,K2,CO4*
9. Define Memory less system. *2,K1,CO5*
10. Check whether the system $y(t) = t\,x(t)$ is linear. *2,K2,CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) A random variable X has the following probability distribution. *16,K3,CO1*

X	0	1	2	3	4	5	6	7
***P*(*x*)**	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find

(i) The value of K.

(ii) $P\,(1.5 < X < 4.5\,/\,X > 2)$ and

(iii)The smallest value of 'n' for which $P(X \leq n) > \frac{1}{2}$.

OR

b) (i) Out of 800 families with 4 children each. 10,K3,CO1

(a) How many families would be expected to have 2 boys & 2 girls?

(b) At least one boy (c) at most 2 girls (d) children of both genders

(ii) State and prove Memoryless property of Exponential distribution. 6,K3,CO1

12. a) The joint probability mass function of (X, Y) is given by $P(x,y) = k(2x+3y), x = 0,1,2;\ y = 1,2,3$. Find all the marginal and conditional probability distributions. Also, find the probability distribution of (X + Y) and also find P $(X + Y > 3)$. 16,K3,CO2

OR

b) The joint probability density function is given by $f(x,y) = \frac{1}{3}(x+y), 0 \leq x \leq 1,\ 0 \leq y \leq 2$. Find the correlation coefficient of X and Y. 16,K3,CO2

13. a) The process $\{X(t)\}$ whose probability distribution under certain condition is given by 16,K3,CO3

$$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2\ldots \\ \frac{at}{1+at}, & n = 0 \end{cases}$$

. Find the mean and variance of the process. Is the process first-order stationary?

OR

b) (i) Examine whether the random process $\{X(t)\} = A\cos(\omega t + \theta)$ is a wide sense stationary if A and ω are constants and θ is uniformly distributed random variable in $(0,2\pi)$. 8,K3,CO3

(ii) The autocorrelation function of a stationary random process is $R(\tau) = 16 + \frac{9}{1+16\tau^2}$. Find the mean and variance of the process. 8,K3,CO3

14. a) (i) Prove that the sum of two independent Poisson processes is a Poisson process. 8,K3,CO4

(ii) A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However, if she buys B and C, the next week she is 3 times as likely to buy A as the other cereal. In the long run how often, she buys each of the 3 cereals? 8,K3,CO4

OR

b) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he 16,K3,CO4 takes a train on the third day and (ii) the probability that he drives to work in the long run.

15. a) A random process X(t) with $R_{XX}(\tau) = e^{-2|\tau|}$ is the output to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \geq 0$. Find (i) Cross correlation $R_{XY}(\tau)$ *16,K3,CO5*

(ii) Cross spectral density $S_{XY}(\omega)$

(iii) Power spectral density $S_{XX}(\omega)$ between the input process X(t) and the output process Y(t).

OR

b) Wide sense stationary process X(t) is the input to a linear system with impulse response $h(t) = 2e^{-7t}, t \geq 0$. If the autocorrelation function of X(t) is $R_{XX}(\tau) = e^{-4|\tau|}$, find the power spectral density of the output process Y(t). *16,K3,CO5*