Reg. No.																

Question Paper Code	12968

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2024

Fourth Semester

Electronics and Communication Engineering

20BSMA401 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

Regulations - 2020

(Use of Statistical Tables is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (20 × 1 = 20 Marks)

Answer ALL Questions

	Question	*Marks*	*K – Level*	*CO*
1.	$\sum_{i=1}^{n} P(x_i)$ is equal to (a) 0 (b) 1 (c) –1 (d) ∞	*1*	*K1*	*CO1*
2.	$Var\ (5x+2)$ is (a) $25\ var\ (x)$ (b) $5\ var\ (x)$ (c) $2\ var\ (x)$ (d) 25	*1*	*K1*	*CO1*
3.	In a binomial distribution, mean is 4 and the variance 3 then the value of p is (a) $3/4$ (b) $1/4$ (c) $4/3$ (d) $2/3$	*1*	*K1*	*CO1*
4.	A continuous random variable x follows the rule $f(x) = Ax^2, 0 < x\ 1$. Determine A? (a) 1 (b) 2 (c) 3 (d) 0	*1*	*K2*	*CO1*
5.	Given a joint probability density function $f(x,y)$ of continuous variables, then the marginal density $f_X(x)$ is given by (a) $f_X(x) = \sum f(x,y)$ (b) $f_X(x) = \int f(x,y)\ dy$ (c) $f_X(x) = \int f(y,x)\,dx$ (d) 0	*1*	*K1*	*CO2*
6.	If *X* and *Y* are two independent random variables then *Cov*(*X*, *Y*) = (a) 0 (b) 1 (c) -1 (d) ∞	*1*	*K1*	*CO2*
7.	If $r_{xy} = 0$ the variables X and Y are (a) linearly related (b) not linearly related (c) independent (d) none of the above	*1*	*K1*	*CO2*
8.	Two random variables X and Y have a covariance of 6. If the variance of Y is9, and the correlation coefficient between X and Y is 0.5, find the variance of X. (a) 16 (b) 15 (c) 10 (d) 12	*1*	*K2*	*CO2*
9.	A random process is defined as: (a) A deterministic function of time (b) A collection of random variables indexed by time or space (c) A process that generates random samples (d) A set of random numbers	*1*	*K1*	*CO3*
10.	The auto-correlation function $R_{XX}(\tau)$ for a random process $\{X(t)\}$ is defined as: (a)$R_{XX}(\tau) = E[X(t)]$ (b) $R_{XX}(\tau) = E[X(t)X(t+\tau)]$ (c) $R_{XX}(\tau) = E[X(t+\tau) - X(t)]$ (d) $R_{XX}(\tau) = Var[X(t)]$	*1*	*K1*	*CO3*
11.	Auto-correlation $R_{XX}(\tau)$ is an ---------- function (a) odd (b) even (c) either odd or even (d) None of the above	*1*	*K1*	*CO3*
12.	The mean square value of the process $E\left(X^2(t)\right)$ is equal to ------ (a) -∞ (b) $R_{XX}(0)$ (c) $R_{XY}(0)$ (d) ∞	*1*	*K1*	*CO3*
13.	Poisson process is a ______ random process. (a) Discrete (b) Continuous (c) Stationary (d) Non-negative	*1*	*K1*	*CO4*
14.	A Markov chain is a Markov process with (a) continuous time and discrete state space. (b) discrete time and discrete state space. (c) discrete time and continuous state space. (d) continuous time and continuous state space.	*1*	*K1*	*CO4*

15. A state is _____ if, when we leave this state, there is a non-zero probability that we will never return to it. (a) irreducible (b) solvable (c) recurrent (d) transient	*1*	*K1*	*CO4*
16. If the number of occurrences of an event A in an interval of length t is a Poisson process X(t) with parameter λ and if each occurrence of A has a constant probability p of being recorded and the recordings are independent of each other, then show that the number N(t) of the recorded occurrences in t is also a Poisson process with parameter -------- (a) np (b) λ (c) λp (d) pq	*1*	*K1*	*CO4*
17. Which of the following is true for a linear system? (a) $f[a_1 X_1(t) \pm a_2 X_2(t)] = a_1 f[X_1(t)] \pm a_2 f[X_2(t)]$ (b) $f[a_1 X_1(t)\, a_2 X_2(t)] = a_1 f[X_1(t)]\, a_2 f[X_2(t)]$ (c) $f[a_1 X_1(t) \pm a_2 X_2(t)] = f[a_1 X_1(t)] \pm f[a_2 X_2(t)]$ (d) None of the above	*1*	*K1*	*CO5*
18. If $Y(t_1)$ depends only on the value of $X(t_1)$ and not on the past or future values of $X(t)$, then the system is (a) linear time invariant (b) casual (c) memoryless (d) constant	*1*	*K1*	*CO5*
19. A linear time-invariant system $y(t) = f(x(t))$ is said to be ------, if its response to any bounded input is bounded. (a) Stable (b) Unstable (c) Bounded (d) Unbounded	*1*	*K1*	*CO5*
20. The power spectral densities of the input and output processes in the system connected by (a) $R_{XX}(\tau) = R_{XX}(\tau) * h(\tau)$ (b) $R_{XX}(\tau) = R_{YX}(\tau) * h(\tau)$ (c) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$ (d) $S_{YY}(w) = S_{XX}(w) * \|H(w)\|^2$	*1*	*K1*	*CO5*

PART - B (10 × 2 = 20 Marks)

Answer ALL Questions

21. If the density function of a continuous random variable X is given by $f(x) = \begin{cases} ax, 0 \le x \le 1 \\ a, 1 \le x \le 2 \\ 3a - ax, 2 \le x \le 3 \\ 0, otherwise \end{cases}$. Find a.	*2*	*K2*	*CO1*
22. The mean and variance of binomial distribution are 5 and 4. Determine the probability distribution.	*2*	*K2*	*CO1*
23. The joint probability mass function of a two-dimensional random variable (*X*, *Y*) is given by $P(x, y) = K(2x + 3y), x = 1, 2;\ y = 1, 2$. Find the value of *K*.	*2*	*K2*	*CO2*
24. Write regression equation of X on Y, and Y on X.	*2*	*K1*	*CO2*
25. Define random process.	*2*	*K1*	*CO3*
26. Find the variance of the stationary process $\{X(t)\}$ whose auto correlation function is given by $R_{XX}(\tau) = 2 + 4e^{-2\|\tau\|}$.	*2*	*K2*	*CO3*
27. Define Gaussian process.	*2*	*K1*	*CO4*
28. State any two properties of a Poisson process.	*2*	*K1*	*CO4*
29. Define memoryless system.	*2*	*K1*	*CO5*
30. If the system has the impulse response $h(t) = \begin{cases} \frac{1}{2c}, \|t\| \le c \\ 0, \quad \|t\| > c \end{cases}$ Write down the relation between the spectrums of input X(t) and output Y(t).	*2*	*K2*	*CO5*

PART - C (6 × 10 = 60 Marks)

Answer ALL Questions

31. a) A discrete random variable X has the following probability distribution *10 K3 CO1*

x	0	1	2	3	4	5	6	7	8
P(x)	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Find the value of 'a'.

(ii) Find $P(0<X<3)$, $P(X \leq 3)$

(iii) Find the distribution function of X

OR

b) Out of 800 families with 4 children each. *10 K3 CO1*

(i) How many families would be expected to have 2 boys & 2 girls?

(ii) At least one boy (iii) at most 2 girls (iv) children of both genders. Assume an equal probability of boys and girls.

32. a) If X and Y are two random variables having joint density function *10 K3 CO2*

$f(x, y) = \frac{1}{8}(6 - x - y); 0 < x < 2, 2 < y < 4$. Find

(i) $P(X < 1 \cap Y < 3)$ (ii) $P(X < 1 \mid Y < 3)$

(iii) $P(X + Y < 3)$.

OR

b) Find the coefficient of correlation between X and Y from the data given below. *10 K3 CO2*

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

33. a) The process $\{X(t)\}$ whose probability distribution under certain condition is given by *10 K3 CO3*

$$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2 \ldots \\ \frac{at}{1+at}, & n = 0 \end{cases}$$

.Find the mean and variance of the process.

Is the process first-order stationary?

OR

b) The auto correlation function of a random telegraph signal process is given by $(\tau) = A^2 e^{-2\alpha|\tau|}$. Determine the PSD of the random telegraph signal. *10 K3 CO3*

34. a) If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$, find the probability that (i) $X(10) \leq 8$, (ii) $|X(10) - X(6)| \leq 4$. *10 K3 CO4*

OR

b) A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However, if she buys B and C, the next week she is 3 times as likely to buy A as the other cereal. In the long run how often, she buys each of the 3 cereals? *10 K3 CO4*

35. a) A random process $X(t)$ with $R_{XX}(\tau) = e^{-2|\tau|}$ is the output to a linear system whose impulse response is $h(t) = 2e^{-t}$, *10 K3 CO5*

$t \geq 0$. Find (i) Cross correlation $R_{XY}(\tau)$

(ii) Cross spectral density $S_{XY}(\omega)$

(iii) Power spectral density $S_{XX}(\omega)$between the input process $X(t)$ and the output process $Y(t)$.

OR

b) A random process $X(t)$ with $R_{XX}(\tau) = e^{-\alpha|\tau|}$, α is a+ve constant, is applied to a input of the linear system whose impulse response is $h(t) = e^{-bt}u(t)$, b is real constant. Find auto correlation of the output process $Y(t)$. *10 K3 CO5*

36.	a) i)	State and prove the memoryless property of Geometric distribution.	*5*	*K3*	*CO1*
	ii)	A lifetime of a certain brand of an electric bulb may be considered as a RV with mean 1200 hours and standard deviation 250 hours. Find the probability, using central limit theorem that the average lifetime of 60 bulbs exceed 1250 hours.	*5*	*K3*	*CO2*
		OR			
	b) i)	A typist types 2 letters erroneously for every 100 letters. What is the probability that the tenth letter typed is the first letter with an error?	*5*	*K3*	*CO1*
	ii)	The joint pdf is given by $f(x,y) = \frac{1}{3}(x+y), 0 \leq x \leq 1, 0 \leq y \leq 2$. Find marginal distributions of X and Y.	*5*	*K3*	*CO2*