Reg. No.															

Question Paper Code	14259

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025

Fourth Semester

Electronics and Communication Engineering

(Common to Computer and Communication Engineering)

20BSMA401 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

Regulations - 2020

(Use of Normal Table is permitted)

Duration: 3 Hours Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

		Marks	K – Level	CO
1.	The random variable X has the following probability distribution: X: 1, 2, 3, 4 $P(X)$: 0.4, 0.3, 0.2, 0.1 Find the mean value of X. (a) 2.0 (b) 2.1 (c) 1.9 (d) 2.5	1	K2	CO1
2.	The moment generating function of a discrete distribution is $(0.6 + 0.4e^t)^6$. The mean value of this distribution is (a) 0.4 (b) 0.6 (c) 0.24 (d) 2.4	1	K2	CO1
3.	If X and Y are two independent random variables, then the correlation coefficient r_{XY} is (a) 1 (b) -1 (c) 0 (d) 0.5	1	K1	CO2
4.	If $Z = X + Y$, where X and Y are independent normal random variables, then the distribution of Z is: (a) Uniform (b) Exponential (c) Normal (d) Binomial	1	K1	CO2
5.	A random process is defined as: (a) A deterministic function of time (b) A collection of random variables indexed by time or space (c) A process that generates random samples (d) A set of random numbers	1	K1	CO3
6.	Let $\{X(t)\}$ and $\{Y(t)\}$ are two random processes with autocorrelation function $R_{XX}(\tau) and\ R_{YY}(\tau)$ respectively and jointly WSS then $\lvert R_{XY}(\tau)\rvert$ is (a) $\leq \sqrt{R_{XX}(0).R_{YY}(0)}$ (b) $\geq \sqrt{R_{XX}(0).R_{YY}(0)}$ (c) $= \sqrt{R_{XX}(0).R_{YY}(0)}$ (d) $< \sqrt{R_{XX}(0).R_{YY}(0)}$	1	K1	CO3
7.	The inter arrival time of a Poisson process with parameter λ is an exponential distribution with mean (a) $\frac{1}{\lambda}$ (b) $-\frac{1}{\lambda}$ (c) $\frac{2}{\lambda}$ (d) λ.	1	K1	CO4
8.	If every state is reachable from every other state, then a Markov chain is said to be (a) Reducible (b) Irreducible (c) Absorbing state (d) Ergodic	1	K1	CO4
9.	The power spectral density of the input and output process in the system are connected by the relation is (a) $S_{xx}(\omega) = \lvert H(\omega)\rvert^2 S_{yy}(\omega)$ (b) $S_{yy}(\omega) = \lvert H(\omega)\rvert^2 S_{xy}(\omega)$ (c) $S_{yy}(\omega) = \lvert H(\omega)\rvert^2 S_{xx}(\omega)$ (d) $S_{xy}(\omega) = \lvert H(\omega)\rvert^2 S_{xx}(\omega)$	1	K1	CO5
10.	Assume a random process $X(t)$ is given as input to a system with transfer function $H(\omega) = 1$ for $-\omega_0 < \omega < \omega_0$. If the auto correlation function of the input process is $N_0\delta(t)/2$, then $S_{YY}(\omega)$ (a) N_0 (b) $N_0/2$ (c) $-N_0$ (d) $-N_0/2$	1	K2	CO5

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

11.	The mean and variance of binomial distribution are 5 and 4. Determine the probability distribution.	2	K2	CO1
12.	A random variable X has the density function $f(x) = \frac{k}{1+x^2}$ if. $-\infty < x < \infty$ Determine the value of k.	2	K2	CO1
13.	State Central limit theorem for independently identically distributed random variables.	2	K1	CO2
14.	Define stationary process.	2	K1	CO2
15.	The autocorrelation function of a stationary random process is $R(\tau) = 16 + \frac{9}{1+16\tau^2}$. Find the mean and variance of the process.	2	K2	CO3
16.	The power spectral density of a random process $\{X(t)\}$ is given by $S_{XX}(\omega) = \begin{cases}\pi, if\ \|\omega\| < 1 \\ 0, elsewhere\end{cases}$. Find its autocorrelation function.	2	K2	CO3
17.	Define Poisson Process.	2	K1	CO3
18.	State any two properties of Gaussian process.	2	K1	CO4
19.	If the initial probability distribution of a Markov Chain is $P^{(0)} = \left(\frac{5}{6}.\frac{1}{6}\right)$ and the transition probability matrix is given as $\begin{bmatrix}0 & 1 \\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$. Find the probability distribution of the chain after 2 steps.	2	K2	CO4
20.	Define memoryless system.	2	K1	CO5
21.	Define unit impulse response system.	2	K1	CO5
22.	Suppose the input $X(t)$ to a linear time invariance system is white noise. Find the power spectral density of the output process $Y(t)$ if the system response $H(\omega)$ is given by $H(\omega) = \begin{cases}1, if\ \omega_1 < \|\omega\| < \omega_2 \\ 0, otherwise\end{cases}$?	2	K2	CO5

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) A random variable X has the following probability distribution. — 11 K3 CO1

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find:

i) The value of K.

ii) $P(1.5 < X < 4.5 / X > 2)$

iii) Evaluate $P(X < 6), P(X \geq 6)$, and $P(0 < X < 5)$

iv) The smallest value of n for which $P(X \leq n) > \frac{1}{2}$

v) Find the distribution function of X.

OR

b) (i) Out of 800 families with 4 children each. How many families would be expected to have (i) 2 boys and 2 girls (ii) At least one boy — 6 K3 CO1

(ii) The number of monthly break-down of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month i) without a break down ii) with only one breakdown iii) with at least one break down. — 5 K3 CO1

24. a) Find the coefficient of correlation between X and Y from the data given below. — 11 K3 CO2

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

OR

b) If X and Y are independent random variables with pdf's $e^{-x}; x \geq 0$ and $e^{-y}; y \geq 0$, respectively, find the density function of $U = \frac{X}{X+Y}, V = X + Y$. Are U and V independent? *11 K3 CO2*

25. a) The process $\{X(t)\}$ whose probability distribution under certain condition is given by $P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2 \ldots \\ \frac{at}{1+at}, & n = 0 \end{cases}$. Find the mean and variance of the process. Is the process first-order stationary? *11 K3 CO3*

OR

b) (i) The auto correlation function of a random telegraph signal process is given by $(\tau) = A^2 e^{-2\alpha|\tau|}$. Determine the Power Spectral Density of the random telegraph signal. *6 K3 CO3*

(ii) Find the Power Spectral Density of a random signal with autocorrelation function $e^{-\lambda|\tau|}$. *5 K3 CO3*

26. a) If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$, find the probability that (i) $X(10) \leq 8$, (ii) $|X(10) - X(6)| \leq 4$. *11 K3 CO4*

OR

b) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run. *11 K3 CO4*

27. a) If{N(t)} is a band limited white noise cantered at carrier frequency ω_0 such that $S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, for\ |\omega - \omega_0| < \omega_B \\ 0, elsewhere \end{cases}$
Find the autocorrelation of $\{N(t)\}$. *11 K3 CO5*

OR

b) If $X(t)$ is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u) du$ *11 K3 CO5*

Then prove that (i) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$
(ii) $R_{YY}(\tau) = R_{XY}(\tau) * h(\tau)$
(iii) $S_{XY}(w) = S_{XX}(w).H^*(w)$
(iv) $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$
$if\ X(t), Y(t)$ are jointly WSS, where * denotes convolution operation.

OR

28. a) (i) A newly constructed township 2000 electric lamps are installed with average life of 1000 burning hours and standard deviation 200 hours. Assuming that the life of lamp follows normal distribution. Find the number of lamps expected fails during first 700 hours. *6 K3 CO1*

(ii) The two lines of regression are $8x - 10y + 66 = 0, 40x - 18y - 214 = 0$. The variance of X is 9. Find (i) mean values of X and Y (ii) correlation coefficient of X and Y. *5 K3 CO2*

OR

b) (i) State and prove the memoryless property of Geometric distribution. *6 K3 CO1*

(ii) If $X_1, X_2, \ldots$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P\ (120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + \cdots X_n$ and $n = 75$. *5 K3 CO2*