Reg. No.																

Question Paper Code	12752

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2024

Fourth Semester

Electronics and Communication Engineering

(Common to Computer and Communication Engineering)

20BSMA401 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

Regulations - 2020

(Use of Statistical Tables is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

	Question	*Marks*	*K – Level*	*CO*
1.	If X and Y are independent random variables with variances 2 and 3. Find the variance of $3X + 4Y$.	*2*	*K2*	*CO1*
2.	Find E(X), if moment generating function of X is $\left(\frac{1}{4}+\frac{3}{4}e^{t}\right)^{5}$	*2*	*K2*	*CO1*
3.	Show that $\frac{2}{5}(2x+3y), 0 \le x \le 1, 0 \le y \le 1$ is a joint pdf of X and Y.	*2*	*K2*	*CO2*
4.	Write regression coefficients.	*2*	*K1*	*CO2*
5.	Define stationary.	*2*	*K1*	*CO3*
6.	The autocorrelation function of a stationary random process is $R(\tau) = 16 + \frac{9}{1+16\tau^2}$. Find the mean and variance of the process.	*2*	*K2*	*CO3*
7.	Define Bernoulli Random Process.	*2*	*K1*	*CO4*
8.	Let A=$\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ be a stochastic matrix. Check whether it is regular.	*2*	*K2*	*CO4*
9.	Define memory less system.	*2*	*K1*	*CO5*
10.	Check whether the system $y(t) = t\,x(t)$ is linear.	*2*	*K2*	*CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) a) A random variable X has the following probability distribution. *16* *K3* *CO1*

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find:

(i). The value of K.

(ii). $P(1.5 < X < 4.5 / X > 2)$.

(iii). Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$.

(iv). The smallest value of n for which $P(X \leq n) > \frac{1}{2}$.

(v). Find the distribution function of X.

OR

b) i) The daily consumption of milk in the city in excess of 20,000 liters is approximately exponential distribution. The average excess in consumption of milk is 3000 litres. The city has a daily stock of 35000 litres. What is the probability that 2 days selected at random the stock is insufficient for both the days. *8 K3 CO1*

ii) A newly constructed township 2000 electric lamps are installed with average life of 1000 burning hours and standard deviation 200 hours. Assuming that the life of lamp follows normal distribution. Find i) Number of lamps expected fails during first 700 hours ii) In what period of burning hours 10% of the lamps failed. *8 K3 CO1*

12. a) The joint probability mass function of (X, Y) is given by $P(x,y) = k(2x+3y), x = 0,1,2;\ y = 1,2,3$. Find all the marginal and conditional probability distributions. Also, find the probability distribution of (X + Y) and ($X + Y > 3$). *16 K3 CO2*

OR

b) Two random variables X and Y have joint density function of *X* and *Y* is *16 K3 CO2*

$f(x,y) = \begin{cases} 2-x-y, 0 \le x \le 1, 0 \le y \le 1 \\ 0, otherwise \end{cases}$. Find Cov (X, Y) and the correlation coefficient between *X* and *Y*.

13. a) i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by *10 K3 CO3*

$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2 \ldots \\ \frac{at}{1+at}, & n = 0 \end{cases}$. Find the mean and variance of the process. Is the process first-order stationary?

ii) The power spectral density of a random process $\{X(t)\}$ is given by $S_{XX}(\omega) = \begin{cases} \pi, if\ |\omega| < 1 \\ 0, elsewhere \end{cases}$. Find its autocorrelation function. *6 K3 CO3*

OR

b) i) The auto correlation function of a random telegraph signal process is given by $R_{XX}(\tau) = A^2 e^{-2\alpha|\tau|}$. Determine the PSD of the random telegraph signal. *8 K3 CO3*

ii) Find the cross-correlation function of the cross-power spectrum of real random process $\{X(t)\}\&\{Y(t)\}$ given by *8 K3 CO3*

$$S_{XY}(\omega) = \begin{cases} a + jb\omega, \text{for} -1 < \omega < 1, \\ 0, \quad elsewhere \end{cases}$$

14. a) If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1,t_2) = 16e^{-|t_1-t_2|}$, find the probability that (i) $X(10) \le 8$, (ii) $|X(10) - X(6)| \le 4$. *16 K3 CO4*

OR

b) i) Prove that the sum of two independent Poisson processes is again a Poisson process. *8 K3 CO4*

ii) A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However, if she buys B and C, the next week she is 3 times as likely to buy A as the other cereal. In the long run how often, she buys each of the 3 cereals? *8 K3 CO4*

15. a) A random process X(t) with $R_{XX}(\tau) = e^{-2|\tau|}$ is the output to a linear system whose impulse response is $h(t) = 2e^{-t}$, t$\geq$ 0. Find (i) Cross correlation $R_{XY}(\tau)$ *16 K3 CO5*

(ii) Cross spectral density $S_{XY}(\omega)$

(iii) Power spectral density $S_{XX}(\omega)$ between the input process x(t) and the output process Y(t).

OR

b) i) If the input to a time-invariant, stable, linear system is WSS process, prove that the output will also be a WSS process. *8 K3 CO5*

ii) Suppose the input X(t) to a linear time invariance system is white noise. Find the power spectral density of the output process Y(t) if the system response $H(\omega)$ is given by $H(\omega) = \begin{cases} 1, & if\ \omega_1 < |\omega| < \omega_2 \\ 0, & otherwise \end{cases}$. *8 K3 CO5*