

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025

Fourth Semester

Computer Science and Engineering

(Common to Information Technology)

20BSMA402 - PROBABILITY AND QUEUEING THEORY

Regulations - 2020

Duration: 3 Hours

Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

- | | <i>Marks</i> | <i>K-
Level</i> | <i>CO</i> |
|---|--------------|---------------------|-----------|
| 1. The parameters of a binomial distribution are $n = 18$ and $p = \frac{1}{3}$, then mean is
(a) 5 (b) 6 (c) 4 (d) 3 | 1 | K2 | CO1 |
| 2. A random variable X is uniformly distributed between 3 and 15. The variance of X is
(a) 10 (b) 8 (c) 12 (d) 6 | 1 | K2 | CO1 |
| 3. If $f(x, y)$ is the joint probability density function of continuous random variables (X, Y), the conditional probability distribution of Y given X is
(a) $f(x) = \frac{f_X(x)}{f_Y(y)}$ (b) $f(x) = \frac{f(x,y)}{f_X(x)}$ (c) $f(x) = \frac{f_Y(y)}{f_X(x)}$ (d) $f(x) = \frac{f(x,y)}{f_Y(y)}$ | 1 | K1 | CO2 |
| 4. The correlation coefficient is used to determine:
(a) A specific value of the y-variable given a specific value of the x-variable
(b) A specific value of the x-variable given a specific value of the y-variable
(c) The strength of the relationship between the x and y variables
(d) None of these | 1 | K1 | CO2 |
| 5. For a Poisson process with parameter λ the time between two successive events follows which distribution?
(a) Uniform distribution (b) Exponential distribution
(c) Normal distribution (d) Gamma distribution | 1 | K1 | CO3 |
| 6. The state transition matrix in a Markov chain:
(a) Contains the probabilities of moving between all pairs of states
(b) Shows the sequence of all visited states
(c) Gives the expected values of all states
(d) Lists all possible states without transition information | 1 | K1 | CO3 |
| 7. In a single-server queue (M/M/1), what does "M" stand for?
(a) Mean service time (b) Memory less arrival process (Markovian)
(c) Maximum number of customers (d) Managerial queue | 1 | K1 | CO4 |
| 8. In an M/G/1 queue, what type of service time distribution is represented by "G"?
(a) Geometric (b) General (c) Gaussian (d) Grouped | 1 | K1 | CO4 |
| 9. In (M/G/1) model, the formula for the average waiting time in the queue is
(a) $\frac{\lambda^2 \sigma^2 + \rho \mu^2}{2[1-\rho]}$ (b) $\frac{\lambda^2 \sigma^2 - \rho^2}{\lambda[1-\rho]}$ (c) $\frac{\lambda^2 \sigma^2 + \rho^2}{2[1-\rho]}$ (d) $\frac{\lambda^2 \sigma^2 + \rho^2}{2\lambda[1-\rho]}$ | 1 | K1 | CO5 |
| 10. The M/M/s queue configuration allows for:
(a) General Service Time (b) Multiple Servers
(c) Constant Service Time (d) A single server | 1 | K1 | CO5 |

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

- | | | | |
|--|---|----|-----|
| 11. State Baye's theorem. | 2 | K1 | CO1 |
| 12. List the Characteristics of Normal distribution. | 2 | K1 | CO1 |
| 13. Let X be a uniformly distributed random variable with mean 1 and variance 4/3. Find $P(X < 0)$. | 2 | K2 | CO1 |

14. Show that the function $f(x, y) = \frac{2}{5}(2x + 3y)$, $0 \leq x \leq 1$, $0 \leq y \leq 1$ is a joint probability density function of X and Y . 2 K2 CO2
15. The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. Find mean values of X and Y . 2 K2 CO2
16. State Central limit theorem. 2 K1 CO2
17. Define: WSS process. 2 K1 CO3
18. Examine whether the Poisson process is stationary or not. 2 K1 CO3
19. Define: Balking of the customers in the queuing system. 2 K1 CO4
20. What is meant by queue discipline? List the various types of queue discipline. 2 K1 CO4
21. What do you mean by series queue with blocking? 2 K1 CO5
22. State Pollaczek- Khintchine formula. 2 K1 CO5

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) A discrete random variable X has the following probability distribution. 11 K3 CO1

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

- (i) Find the value of 'a'
(ii) Find $P(X < 3)$, $P(0 < X < 3)$, $P(X \geq 3)$
(iii) Find the distribution function of X .

OR

- b) Find the moment generating function of Binomial distribution function and hence find mean and variance. 11 K3 CO1

24. a) The joint probability mass function of (X, Y) is given by $P(x, y) = K(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal and conditional probability distributions. 11 K3 CO2

OR

- b) Calculate the correlation coefficient for the following heights in inches of fathers (X) and their sons (y). 11 K3 CO2

X	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

25. a) Two boys B_1, B_2 and two girls G_1, G_2 are throwing a ball from one to another. Each boy throws the ball to the other boy with probability $\frac{1}{2}$ and to each girl with probability $\frac{1}{4}$. On the other hand, each girl throws the ball to each boy with probability $\frac{1}{2}$ and never to the other girl. In the long run, how often does each receive the ball? 11 K3 CO3

OR

- b) Prove that inter arrival time between two successive occurrences of Poisson process follows Exponential distribution. 11 K3 CO3

26. a) A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and car arrive for service in a Poisson process at the rate of 30 cars per hour. 11 K3 CO4
- (i) What is the probability that an arrival would have to wait in line?
(ii) Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system?
(iii) For what % of time would a pump be idle on an average?

OR

- b) A two-person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min. in the barber's chair. Compute $P_0, P_1, P_7, E(N_q)$ and $E(W)$. 11 K3 CO4

27. a) A one-man barbershop takes 25 minutes to complete one haircut. If customers arrive at the barbershop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service. 11 K3 CO5

OR

- b) Derive Pollaczek – Khinchine formula. 11 K3 CO5

28. a) (i) A supermarket has a single cashier. During peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate the average number of customers in the queuing system. 6 K3 CO4
(ii) An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes then determine L_s . 5 K3 CO5

OR

- b) (i) There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. What fraction of the time all the typists will be busy? 6 K3 CO4
(ii) Jackson network with three facilities that have the parameters given below 5 K3 CO5
 $P_{11} = 0, P_{12} = 0.6, P_{13} = 0.3, P_{21} = 0.1, P_{22} = 0, P_{23} = 0.3,$
 $P_{31} = 0.4, P_{32} = 0.4, P_{33} = 0, \mu_1 = 10, \mu_2 = 10, \mu_3 = 10, c_1 = 1,$
 $c_2 = 2, c_3 = 1, r_1 = 1, r_2 = 4, r_3 = 3.$
Find the total arrival rate at each facility $P(n_1, n_2, n_3)$.