Reg. No.												

Question Paper Code	12414

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2023

Fourth Semester

Computer Science and Engineering

(Common to Information Technology & M.Tech. - Computer Science and Engineering)

20BSMA402 - PROBABILITY AND QUEUING THEORY

(Use of Statistical Table is permitted)

(Regulations 2020)

Duration: 3 Hours Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

Marks, K-Level, CO

1. State Baye's theorem. *2,K1,CO1*
2. Ten coins are thrown simultaneously. Find the probability of getting at least seven heads. *2,K2,CO1*
3. If the random variables X & Y has the joint probability density function $f(x, y) = kx^2 (8 - y), 0 < x < 1, x < y < 1$, find value of k. *2,K2,CO2*
4. State central limit theorem. *2,K1,CO2*
5. Define weak sense stationary. *2,K1,CO3*
6. Define Markov chain. *2,K1,CO3*
7. Explain Kendall's notation. *2,K1,CO4*
8. If there are 2 servers in an infinite capacity Poisson queue system with $\lambda = 10$ per hour and $\mu = 15$ per hour, what is the percentage of idle time for each server? *2,K2,CO4*
9. Define Open Jackson Networks. *2,K1,CO5*
10. What is a series queue with Blocking? *2,K1,CO5*

PART - B (5 × 16 = 80 Marks)

Answer ALL Questions

11. a) (i) The contents of urns I, II and III are as follows: *8,K3,CO1*

	White balls	Black balls	Red balls
Urn I	1	2	3
Urn II	2	1	1
Urn III	4	5	3

One urn is chosen at random and two balls are drawn. What is the probability that the balls are turn to be white and red?

(ii) The density function of a continuous random variable X is given by 8,K3,CO1

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0 & otherwise \end{cases}$$

(a) Find the value of 'a' (b) Distribution function of X.

OR

b) A random variable X has the following probability function 16,K3,CO1

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+ K

Find:

(a) value of K.
(b) P (X < 6), P (X ≥ 6), P (0 < X < 5).
(c) the distribution function of X.

12. a) If the joint p.d.f of a two dimensional RV (x, y) is given by 16,K3,CO2

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3} & ; 0 < x < 1, 0 < y < 2. \\ 0 & ; \text{Elsewhere.} \end{cases}$$

Then find

(a) Marginal density functions of X & Y,
(b) $P(Y < \frac{1}{2} / X < \frac{1}{2})$
(c) Check whether X and Y are independent.

OR

b) Find the equation of two regression lines for the following data. Hence find X when Y = 66 8,K3,CO2

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

13. a) The process $\{X(t)\}$ whose probability distribution under certain condition is given by $P[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+a\ )^{n+1}}, & n = 1,2,3,... \\ \frac{at}{1+at}, & n = 0 \end{cases}$. 16,K3,CO3

Show that $\{X(t)\}$ is not stationary

OR

b) (i) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day, he sells in city B. However, if he sells in either B or C, the next day he is twice as likely to sell in city A as in the other city. In the long run, how often does he sell in each of the cities? 8,K3,CO3

(ii) Consider a random process Y (t) = X (t) cos (ω_0t + θ) where X (t) is wide sense stationary random process. θ is a random variable independent of X(t) and is distributed uniformly in ($-\pi, \pi$) and ω_0 is a constant. Prove that Y(t) is wide-sense stationary. *8,K3,CO3*

14. a) There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, *16,K3,CO4*
(a) What fraction of the time all the typists will be busy?
(b) What is the average number of letters waiting to be typed?
(c) What is the average time a letter has to spend for waiting and for being typed?

OR

b) Customers arrive at a one-man barber shop according to Poisson process with mean interval time 20 minutes. Customers spend an average of 15 minutes in the barber chair. If an hour is used as the unit of time, then *16,K3,CO4*
(a) Find the probability that the customer has to wait for the service.
(b) What is the probability that a customer need not wait for a haircut?
(c) What is the expected number of customers in the barber shop?
(d) What is the expected number of customers in the queue?

15. a) Derive the Pollaczek – Khintchine formulae for the M / G/ 1 queuing model. *16,K3,CO5*

OR

b) (i) Write a short note on Open and Closed Jackson Network. *6,K3,CO5*

(ii) Consider a system of two servers where customers from outside the system arrive at server 1 at a poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to server 2or leave the system, whereas a departure from server 2 will go 25 percent of the time to server 1and will depart the system otherwise. Determine the limiting probabilities, L_S and W_S . *10,K3,CO5*