Reg. No.																

Question Paper Code	12753

B.E. / B.Tech./ M .Tech - DEGREE EXAMINATIONS, APRIL / MAY 2024

Fourth Semester

Computer Science Engineering

(Common to Information Technology and M .Tech - Computer Science and Engineering)

20BSMA402 – PROBABILITY AND QUEUING THEORY

Regulations - 2020

(Use of *Normal distribution table* is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
1.	Let A and B be two events such that $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{4}$ and $P(A\cap B)=\frac{1}{4}$. Compute $P\left(A/B\right)$ and $P\left(\overline{A}\cap\overline{B}\right)$.	2	K2	CO1
2.	If the moment generating function of a random variable X is of the form $\left(0.4e^{t}+0.6\right)^{8}$. Evaluate $E(X)$.	2	K2	CO1
3.	Find the value of k, if the joint density function of (X,Y) is given by $f(x,y)=\begin{cases} k(1-x)(1-y); & 0<x<4, 1<y<5 \\ 0; & \text{otherwise} \end{cases}$.	2	K3	CO2
4.	The two regression equations of the variables X and Y are $x=20.1-0.5y$ and $y=11.64-0.8x$. Find the means of X and Y.	2	K3	CO2
5.	Write the classification of random processes.	2	K1	CO3
6.	Consider a Markov chain with state {0, 1, 2} and transition probability matrix $P=\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{pmatrix}$. Draw the state transition diagram.	2	K2	CO3
7.	What do the letters in the symbolic representation (*a*/*b*/*c*) (*d*/*e*) of a queueing model represent?	2	K1	CO4
8.	What do you mean by balking, reneging of a queueing system?	2	K1	CO4
9.	Define series queue model.	2	K1	CO5
10.	State Jackson's theorem for an open network.	2	K1	CO5

PART - B (5 ×16 = 80 Marks)

Answer ALL Questions

11. a) i) A random variable X has the following probability distribution: *8 K3 CO1*

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find :

(1) The value of k.

(2) P(1.5< X < 4.5/ X>2)

(3) The cumulative distribution function of X.

ii) The number of typing mistakes that a typist makes on a given page has a Poisson distribution with a mean of 3 mistakes. What is the probability that she makes *8 K3 CO1*

(1) Exactly 7 mistakes

(2) Fewer than 4 mistakes

(3) No mistakes on a given page.

OR

b) i) The distribution function of a random variable X is given by $F(x)=1-(1+x)e^{-x}, x\geq 0$. Find the density function, mean and variance of$X$. *8 K3 CO1*

ii) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda=\frac{1}{2}$. *8 K3 CO1*

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the conditional probability that a repair time takes at least 10 hours given that its duration exceeds 9 hours?

12. a) i) Two random variables X and Y have the joint probability density function $f(x,y)=\begin{cases}2-x-y, & 0\leq x\leq 1, 0\leq y\leq 1\\ 0, & \text{otherwise}\end{cases}$. Find the marginal densities of X and Y. Also, find the conditional density functions. *8 K3 CO2*

ii) If the joint pdf of (X,Y) is given by $f(x,y)=x+y$, $0\leq x,y\leq 1$, find the pdf of the R.V. $U=XY$. *8 K3 CO2*

OR

b) i) Calculate the correlation coefficient for the following data: *8 K3 CO2*

X:	55	56	58	59	60	60	62
Y:	35	38	37	39	44	43	44

ii) Let $X_1, X_2, ..., X_{100}$ be independent identically distributed random variables with $\mu=2$ and $\sigma^2=\frac{1}{4}$. Find $P(192<X_1+X_2+...+X_{100}<210)$. *8 K3 CO2*

13. a) i) A random process $\{X(t)\}$ has the probability distribution $P[X(t)=x]=\begin{cases}\dfrac{(at)^{x-1}}{(1+at)^{x+1}}, & x=1,2,3,...\\ \dfrac{at}{1+at}, & x=0\end{cases}$. Show that the process is not stationary. 8 K3 CO3

ii) Let $\{X_n\}$ be a Markov chain with state space {1, 2, 3} with initial probability vector $P^{(0)}=(0.7,0.2,0.1)$ and the one step transition probability matrix $P=\begin{pmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{pmatrix}$. Compute $P(X_2=3)$ and $P(X_3=2,X_2=3,X_1=3,X_0=2)$. 8 K3 CO3

OR

b) i) An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follows a highly distorted signal, with no recognizable signals between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted. 8 K3 CO3

ii) Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute; find the probability that during a time interval of 2 minutes (1) exactly 4 customers arrive and (2) more than 4 customers arrive. 8 K3 CO3

14. a) Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then 16 K3 CO4

(i) What is the probability that a customer need not wait for a haircut?

(ii) What is the expected number of customer in the barber shop?

(iii) What is the expected number of customer in the queue?

(iv) How much time can a customer expect to spend in the barber shop?

(v) Find the average time that a customer spend in the queue.

(vi) Estimate the fraction of the day that the server will be idle?

(vii) What is the probability that there will be 6 or more customers?

OR

b) i) A supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, what is the probability that a customers has to wait for service? What is the expected percentage of idle time for each girl? If the customer has to wait in the queue, what is the expected length of his waiting time? *8 K3 CO4*

ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. *8 K3 CO4*

(1) Find the effective arrival rate at the clinic.

(2) What is the probability that an arriving patient will not wait?

(3) What is the expected waiting time until a patient is discharged from the clinic?

15. a) Derive the Pollaczek-Khinchine formula for the average number in the system in a M/G/1 queueing model. *16 K3 CO5*

OR

b) Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s. *16 K3 CO5*