Reg. No.															

Question Paper Code	12755

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2024

Fourth Semester

Computer Science and Engineering (AIML)

(Common to Artificial Intelligence and Data Science)

20BSMA404 - LINEAR ALGEBRA AND ITS APPLICATIONS

Regulations - 2020

Duration: 3 Hours | Max. Marks: 100

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
1.	Can Vector $\begin{bmatrix}3\\2\end{bmatrix}$ be written as the linear combination of vectors $\begin{bmatrix}-3\\1\end{bmatrix}$ and $\begin{bmatrix}0\\-2\end{bmatrix}$?	2	K1	CO1
2.	Find the rank of $\begin{pmatrix}1 & 1 & 0 & -2\\ 2 & 0 & 2 & 2\\ 4 & 1 & 3 & 1\end{pmatrix}$.	2	K1	CO1
3.	Find transpose and trace of $\begin{pmatrix}-4 & 2\\ 5 & -1\end{pmatrix}$.	2	K1	CO2
4.	Define Basis and Dimension.	2	K1	CO2
5.	If $T: R^2 \to R^2$ by $T(a_1, a_2) = (2a_1 + a_2, a_1)$ then verify T is Linear or not.	2	K1	CO3
6.	Let $V(F)$ and $W(F)$ be vector spaces and $T: V \to W$ be a linear transformation. Then $prove\ that\ N(T)$ is a subspace of V.	2	K2	CO3
7.	Define Orthonormal set.	2	K1	CO4
8.	Show that $\|x+y\|^2 + \|x-y\|^2 = \|x\|^2 + \|y\|^2$ for x, y ∈V.	2	K2	CO4
9.	Find the largest eigenvector of $A = \begin{pmatrix}4 & 0\\ 3 & -5\end{pmatrix}$.	2	K1	CO5
10.	State any two applications of PCA in data science.	2	K1	CO5

PART - B (5 ×16 = 80 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
11. a) i)	Determine the value of a and b if the matrix $A = \begin{pmatrix}2 & 1 & -1 & 3\\ 1 & -1 & 2 & 4\\ 7 & -1 & a & b\end{pmatrix}$ is of rank 2.	6	K3	CO1
ii)	Solve the System of equations by Gaussian Elimination Method $2x - 2y + 3z = 2;$ $x + 2y - z = 3;$ $3x - y + 2z = 1$	10	K3	CO1

OR

b) Solve the system of equations $x_1 + x_2 + x_3 = 1;\ 3x_1 + x_2 - 3x_3 = 5;\ x_1 - 2x_2 - 5x_3 = 10$ by LU decomposition method. *16 K3 CO1*

12. a) i) Prove that $2x^3 - 2x^2 + 12x - 6$ is a linear combination of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$. *8 K3 CO2*

ii) Verify the set $\{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\}$ in $P_3(R)$ is linearly independent or linearly dependent. *8 K3 CO2*

OR

b) i) Let V beavector space and $\beta = \{u_1, u_2, \ldots . u_n\}$ be a subset of V. Then prove that β is a basis for V if and only if each $v \in V$ can be uniquely expressed as a linear combination of vectors of β. *8 K3 CO2*

ii) Prove that a subset W of a vector space V is a subspace of V if and only if $\alpha u + \beta v \in W$ for all u,v $\in W$ and $\alpha, \beta \in F$. *8 K3 CO2*

13. a) i) Let V and W be vector spaces and let $T: V \to W$ be linear. If V is finite-dimensional then prove that $Nullity(T) + Rank\ (T) = \dim(V)$. *10 K3 CO3*

ii) Prove that there exists a linear transformation T:$R^2 \to R^3$ such that T (1,1) = (1,0,2) and T (2,3) = (1, -1,4). What is T (8,11). *6 K3 CO3*

OR

b) For the following matrix $A \in M_{n\times n}(R)$, test A for diagonalizability and if A is diagonalizable, find an invertible matrix Q and a diagonal matrix D such that$Q^{-1}AQ = D$. *16 K3 CO3*

$$A = \begin{pmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ 6 & -6 & 3 \end{pmatrix}$$

14. a) Find QR Decomposition (Gram Schmidt Method) for the following Matrix. *16 K3 CO4*

$$\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

OR

b) Let $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$ Find the orthogonal matrix P and the diagonal matrix D such that $P^tAP = D$. *16 K3 CO4*

15. a) Find Singular Value Decomposition for $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$. *16 K3 CO5*

OR

b) Suppose A_0 has these two measurements of 5 sample. $A_0 = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix}$. Compute the centred matrix A, sample covariance S, eigen values λ_1, λ_2. what is the line through the origin is closest to the 5 sample in the column of A. *16 K3 CO5*