Reg. No.																

Question Paper Code	12971

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2024

Fourth Semester

Artificial Intelligence and Data Science

(Common to Computer Science and Engineering (AIML))

20BSMA404 - LINEAR ALGEBRA AND ITS APPLICATIONS

Regulation - 2020

Duration: 3 Hours   Max. Marks: 100

PART - A (MCQ) (20 × 1 = 20 Marks)

Answer ALL Questions

	Question	Marks	K – Level	CO
1.	Which of the following set of vectors is linearly dependent? (a) (1,0,1),(-1,1,0),(5,-1,2) (b) (1,2,0),(1,1,1),(2,0,1) (c) (2,3,-1),(-4,2,-6),(5,-4,9) (d) (2,0,2),(-1,1,0),(10,-1,2)	1	K2	CO1
2.	Which of the following set of vectors in R^3 in linearly independent R^3? (a){(1,2,5),(1,-2,1),(2,1,4)} (b){(1,-2,3),(-2,4,1),(-4,8,9)} (c) {(2,-4,6),(-2,4,1),(-4,8,9)} (d) {(2,-1,3),(-4,2,-6),(8,0,1)}	1	K2	CO1
3.	The rank of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -4 \\ 1 & 1 & -1 \end{bmatrix}$ is (a) 2 (b) 3 (c) 4 (d) 1	1	K2	CO1
4.	The goal of forward elimination steps in the Gauss elimination method is to reduce the coefficient matrix to a (an) _____________ matrix. (a) diagonal (b) identity (c) lower triangular (d) upper triangular	1	K1	CO1
5.	If the system of linear equations Ax = b has a solution, then b is: (a) Always a linear combination of the columns of A. (b) Never a linear combination of the columns of A. (c) A linear combination of the rows of A. (d) Always orthogonal to the columns of A.	1	K1	CO2
6.	Which of the following is not a property of a vector space? (a) The zero vector is an element of the space. (b) Vector addition is commutative. (c) Every vector has an inverse with respect to addition. (d) Scalar multiplication is not distributive over scalar addition.	1	K1	CO2
7.	Which of the following is true for a subspace W of a vector space V? (a) W can contain vectors that are not in V. (b) W must contain the zero vector of V. (c) The span of W is V. (d) If W is a subspace of V, then Vis a subspace of W.	1	K1	CO2
8.	If a vector space V has a finite basis with n vectors, then the dimension of V is: (a) Equal to n. (b) Less than n. (c) Greater than n. (d) Undefined.	1	K1	CO2
9.	The eigen values of a triangular matrix are the __________ elements of the matrix (a) 1st Row (b) 1st Column (c) Diagonal (d) any column	1	K1	CO3

10.	Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined as $T(x, y) = (\sin x, 0)$. Then (a) T is Linear (b) T is not linear (c) Constant (d) none of these	*1*	*K2*	*CO3*
11.	The matrix of the Q.F $10x_1^2 + 2x_2^2 + 4x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$ is _____ (a) $A = \begin{pmatrix} 10 & -2 & 5 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ (c) $A = \begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 4 \end{pmatrix}$ (d) $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$	*1*	*K2*	*CO3*
12.	The nature of the quadratic form $2xy + 2yz + 2zx$ is _______. (a) Indefinite (b) +ve semi-definite (c) +ve definite (d) –ve definite	*1*	*K1*	*CO3*
13.	Which of the following is not a requirement for a space to be an inner product space? (a) The space is a vector space over the real or complex numbers. (b) The inner product satisfies linearity in both arguments. (c) The inner product is symmetric or conjugate symmetric. (d) The inner product satisfies the triangle inequality.	*1*	*K1*	*CO4*
14.	Which of the following is true about orthogonal vectors in an inner product space? (a) They must have the same norm. (b) Their inner product is zero. (c) They are linearly dependent. (d) They can only be in R^2.	*1*	*K1*	*CO4*
15.	The least squares approximation is used to solve: (a) Over determined systems of linear equations (more equations than unknowns). (b) Underdetermined systems of linear equations (more unknowns than equations). (c) Systems with exactly as many equations as unknowns. (d) Systems that are consistent and have a unique solution.	*1*	*K1*	*CO4*
16.	The Gram-Schmidt orthogonalization process is used to: (a) Convert a set of linearly independent vectors into an orthonormal set. (b) Convert a set of linearly dependent vectors into an orthonormal set. (c) Find the null space of a matrix. (d) Solve a system of linear equations.	*1*	*K1*	*CO4*
17.	The matrix Σ in the Singular Value Decomposition (SVD) of A contains: (a) The eigenvalues of A. (b) The singular values of A on its diagonal. (c) The eigenvectors of A. (d) The columns of A.	*1*	*K1*	*CO5*
18.	The rank of the matrix A is equal to: (a) The number of non-zero entries in Σ. (b) The number of rows in A. (c) The number of columns in A. (d) The number of columns in V.	*1*	*K1*	*CO5*
19.	PCA is widely used in which of the following areas of data science? (a) Time series prediction. (b) Data visualization and feature extraction. (c) Predicting labels in supervised learning. (d) Model evaluation and validation.	*1*	*K1*	*CO5*

20.	The main advantage of using PCA for dimensionality reduction in large datasets is: (a) PCA can handle missing data automatically. (b) PCA is computationally intensive, making it suitable for small datasets. (c) PCA reduces the size of the dataset while preserving most of the variance. (d) PCA can automatically detect outliers in the data.	*1*	*K1*	*CO5*

PART - B (10 × 2 = 20 Marks)

Answer ALL Questions

21.	Define Gauss elimination method.	*2*	*K1*	*CO1*
22.	Vector (1, K, 5) is a linear combination of (1, -3, 2) and (2, -1, 1). Find K.	*2*	*K2*	*CO1*
23.	Consider $W = \{(x_1, x_2, x_3)/x_1 = x_3 + 2\}$. Prove that W is not a subspace of V.	*2*	*K2*	*CO2*
24.	Define Linearly Independent vectors.	*2*	*K1*	*CO2*
25.	Define Null space.	*2*	K1	*CO3*
26.	If $T: R \to R$ is defined by $T(x) = 2^x, \forall x \epsilon R$, show that T is not linear.	*2*	*K2*	*CO3*
27.	State Inner Product spaces.	*2*	*K1*	*CO4*
28.	If $< x, y > = < x, z >$ for all x ∈ V, then y = z.	*2*	*K2*	*CO4*
29.	What is total Variance?	*2*	*K1*	*CO5*
30.	Define Singular Value Decomposition (SVD).	*2*	*K1*	*CO5*

PART - C (6 × 10 = 60 Marks)

Answer ALL Questions

31.	a)	Solve by LU Decomposition method. $x - 3y - 8z = -10, \;\; 3x + y = 4z, \; 2x + 5y + 6z = 13.$	*10*	*K3*	*CO1*
		OR			
	b)	Solve the system by Gauss Elimination method. $x - y + z = 1; \;\; -3x + 2y - 3z = -6; \quad 2x - 5y + 4z = 5$	*10*	*K3*	*CO1*
32.	a)	Determine whether the set $W = \{(1,0,-1), (2,5,1), (0,-4,3)\} \subseteq R^3$ is a basis of R^3	*10*	*K3*	*CO2*
		OR			
	b)	Prove that the set of all $m \times n$ matrices over F is denoted by $M_{m\times n}(F)$ a vector space over a field F operation of matrix addition and scalar multiplication of matrix.	*10*	*K3*	*CO2*
33.	a)	Find the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$	*10*	*K3*	*CO3*
		OR			
	b)	State and prove Dimension Theorem.	*10*	*K3*	*CO3*
34.	a)	State and prove Cauchy-Schwarz inequality.	*10*	*K3*	*CO4*
		OR			

	b)	Determine the QR-Decomposition of the matrix A = $\begin{bmatrix} 2 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 2 & 4 \end{bmatrix}$	*10*	*K3*	*CO4*
35.	a)	Determine the matrix U, $\sum V$ such that A = U$\sum V^T$, Where A = $\begin{bmatrix} 6 & 0 \\ 8 & 10 \end{bmatrix}$	*10*	*K3*	*CO5*

OR

	b)	Compute $A^T A$ and AA^T, their eigen values and eigen vector v and u for the rectangular matrix A = $\begin{bmatrix} 4 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and compute SVD of the matrix A = U.	*10*	*K3*	*CO5*
36.	a) i)	Compute the least square solution of the equations $2x + 10y = 6$, $4x - 4y = 4$, $2x - 2y = -10$.	*5*	*K3*	*CO4*
	ii)	Discuss the applications of PCA in Data Science.	*5*	*K3*	*CO5*

OR

	b) i)	Let A = $\begin{pmatrix} 1 & 2+i \\ 3 & i \end{pmatrix}$ and B = $\begin{pmatrix} 1+i & 0 \\ i & -i \end{pmatrix}$. Use the Frobeninus inner product on M $_{nxn}$(F) and compute ‖A‖, ‖B‖,.	*5*	*K3*	*CO4*
	ii)	What are the applications of SVD in Data Science?	*5*	*K3*	*CO5*