Reg. No.												

Question Paper Code	12416

B.E. / B.Tech - DEGREE EXAMINATIONS, NOV / DEC 2023

Fourth Semester

Artificial Intelligence and Data Science

20BSMA404 - LINEAR ALGEBRA AND ITS APPLICATIONS

(Regulations 2020)

Duration: 3 Hours | Max. Marks: 100

PART - A ($10 \times 2 = 20$ Marks)

Answer ALL Questions

		Marks, K-Level, CO
1.	Express the vector $(1,-2,5)$ as a linear combination of vectors $(1,1,1),(1,2,3)\&(2,-1,1)$.	*2,K1,CO1*
2.	Define Rank of a Matrix.	*2,K1,CO1*
3.	Define subspace with example.	*2,K2,CO2*
4.	Write the standard basis of the vector space $P_n(R)$.	*2,K2,CO2*
5.	Let $T:R^2 \rightarrow R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_1)$. Show that T is linear.	*2,K1,CO3*
6.	Let $V(F)$ and W (F) be vector spaces and $T: V \rightarrow W$ be a linear transformation. Then $N(T)$ is a subspace of V.	*2,K1,CO3*
7.	Prove that $< x, cy > = \bar{c} < x, y >$.	*2,K1,CO4*
8.	Let V be an inner product space, and suppose that x and y are orthogonal vectors in V. prove that $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.	*2,K2,CO4*
9.	Define Singular Vector.	*2,K2,CO5*
10.	Why we use SVD and PCA in Data Science.	*2,K2,CO5*

PART - B ($5 \times 16 = 80$ Marks)

Answer ALL Questions

11. a) (i) Find the value of a and b if $A = \begin{pmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 4 \\ 7 & -1 & a & b \end{pmatrix}$ is of rank 2. *8,K2,CO1*

(ii) Solve by Cramer's rule *8,K3,CO1*

$$x_1 + x_2 + x_3 = 11$$
$$2x_1 - 6x_2 - x_3 = 0$$
$$3x_1 + 4x_2 + 2x_3 = 0$$

OR

b) (i) Solve by LU Decomposition method *8,K3,CO1*

$$x - 3y - 8z = -10$$
$$3x + y = 4z$$
$$2x + 5y + 6z = 13$$

(ii) Solve the system by Gauss Elimination Method 8,K3,CO1

$$5x_1 + 3x_2 + 7x_3 = 4$$
$$3x_1 + 26x_2 + 2x_3 = 9$$
$$7x_1 + 2x_2 + 10x_3 = 5$$

12. a) Show that $R^n = \{(x_1, x_2, x_3, \dots, x_n): x_i \in R\}$is a vector space over F with respect to addition and scalar multiplication defined component wise. 16,K3,CO2

OR

b) (i) Prove that the intersection of two subspaces of a vector space V is again a subspace of V. 8,K3,CO2

(ii) The union of two subspaces a vector space is a subspace of V if f one is contains the other. 8,K3,CO2

13. a) (i) Let $T: R^3 \to R^2$ be linear transformation defined by $T(x, y, z) = (x - y, 2z)$. find N(T), R(T), nullity and rank of T. 8,K3,CO3

(ii) Determine the matrix of the linear transformation $T: R^2 \to R^3$ defined by $T(x, y) = (x - y, x, 2x + y)$ relative to the standard basis R^2. 8,K3,CO3

OR

b) State and prove Dimension Theorem. 16,K3,CO3

14. a) (i) State and prove Cauchy-Schwarz inequality. 8,K4,CO4

(ii) Compute the least square solution of the equations x+5y=3; 2x-2y =2; -x +y =5 . Also find the least square error. 8,K4,CO4

OR

b) Determine the QR-Decomposition of the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$. 16,K4,CO4

15. a) (i) Determine the matrix U, Σ, V such that A=UΣV^T, where A=$\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$. 8,K4,CO5

(ii) Discuss the applications of Linear Algebra in Data Science. 8,K4,CO5

OR

b) Suppose A_0 has these two measurements of 6 sample. A_0=$\begin{bmatrix} 1 & 0 & -1 & 1 & 2 & 3 \\ 0 & 1 & 0 & 3 & 2 & 1 \end{bmatrix}$ Compute the centred matrix A, sample covariance S, eigen values λ_1, λ_2. what is the line through the origin is closest to the 6 sample in the column of A. 16,K4,CO5