Reg. No.																

Question Paper Code	13888

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025

Third Semester

Computer Science and Business Systems

24BSMA305 - LINEAR ALGEBRA

Regulations - 2024

Duration: 3 Hours — Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
1.	The inverse of the matrix $S = \begin{pmatrix} 1 & -2 \\ 0 & a \end{pmatrix}$ exists if (a) $a = 0$ (b) only when $a = 1$ (c) $a \neq 0$ (d) for all values of a	1	K2	CO1
2.	The determinant of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ is (a) – 2 (b) 0 (c) 1 (d) 3	1	K2	CO1
3.	The rank of the matrix $A = \begin{pmatrix} 7 & -2 \\ 3 & 4 \end{pmatrix}$ is (a) – 1 (b) 0 (c) 1 (d) 2	1	K2	CO2
4.	If $rank(A) = rank(A, B) <$ number of unknowns, then the system of equations has (a) No solution (b) A unique solution (c) Infinitely many solutions (d) Exactly two solutions	1	K1	CO2
5.	The dimension of the vector space RxR is (a) 1 (b) 2 (c) 3 (d) 4	1	K2	CO3
6.	Which of the following sets of vectors is linearly independent in the vector space R^2? (a) {(1,2),(3,6)} (b) {(0,-1),(3,0)} (c) {(4,0),(0,1),(1,1)} (d) {(1,1),(0,0),(3,3)}	1	K2	CO3
7.	Consider the inner product space R^4 with standard inner product. If vector $u = (3, 2, k, -5)$ and $v = (1, k, 7, 3)$ are orthogonal. Then the value of k is (a) $\frac{3}{4}$ (b) $\sqrt{\frac{3}{4}}$ (c) $\frac{4}{3}$ (d) $\sqrt{\frac{4}{3}}$.	1	K2	CO4
8.	In R^2, with the standard inner product, the length of $u = (3,4)$ is (a) 5 (b) 7 (c) 25 (d) 12	1	K2	CO4
9.	The matrix representing the linear transformation $\boldsymbol{T}: \boldsymbol{P_2(R)} \to \boldsymbol{P_2(R)}$ defined by $\boldsymbol{T(f(x)) = f'(x)}$, with respect to the standard basis $\{\boldsymbol{1, x, x^2}\}$. (a) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	1	K2	CO5
10.	The objective of Principal Component Analysis is (a) To identify improper dataset (b) To increase dimensionality (c) To reduce dimensionality while retaining variance (d) To perform correlation analysis	1	K1	CO6

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
11.	What is a symmetric matrix? Give an example.	2	K1	CO1
12.	Find the inverse of the matrix $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$.	2	K2	CO1

No.	Question	Marks	K	CO
13.	What is meant by LU Decomposition?	2	K1	CO2
14.	Solve the system of linear equations $x - y = -1, 3x + 2y = 12$.	2	K2	CO2
15.	Define subspace of a vector space.	2	K1	CO3
16.	What is the dimension of the vector space C over R ?	2	K1	CO3
17.	In R^3, the Euclidean inner product defined as $< u,v > = 2u_1v_1 + 3u_2v_2$. Let $u = (1,1), v = (3,2)$ and $w = (0,-1)$. Compute the value of $< u - v, 2w >$.	2	K2	CO4
18.	Let V = R^2 and $S = \{(1,0),(0,1)\}$. Verify whether San orthonormal basis is or not.	2	K2	CO4
19.	Let $T: R \to R$ by $T(x) = x + 6, \forall x \in R$ .Verify that T is linear or not.	2	K2	CO5
20.	State the Dimension theorem on Linear transformation.	2	K1	CO5
21.	What is singular value decomposition?	2	K1	CO6
22.	Where principal component analysis is applied in image processing?	2	K1	CO6

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) (i) Solve the equations $x + y + z = 5;\ x - 2y - 3z = -1;\ 2x + y - z = 3$ using Cramer's Rule. *5 K3 CO1*

(ii) Find the rank of the matrix $\begin{pmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{pmatrix}$ using matrix tools. *6 K3 CO1*

OR

b) (i) Solve the linear equation $AX = B$ where $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \\ 2 & 6 & 13 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$ by finding A^{-1}. *6 K3 CO1*

(ii) Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$=2abc$(a+b+c)^3$. *5 K3 CO1*

24. a) Solve the system of equations $x + 2y + 3z = 7; 2x + y - 2z = 1;$ $x - 4y + z = 7$ by Gaussian elimination method. *11 K3 CO2*

OR

b) Calculate the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 6 & -3 \\ 3 & 10 & -5 \end{pmatrix}$. *11 K3 CO2*

25. a) Let R^+ be the set of all positive real numbers. Define addition and scalar multiplication as follows: $u + v = uv$ for all $u, v \in R^+$; $\alpha . u = u^{\alpha}$ for all $u \in R^+$ and $\alpha \in R$. Determine whether or not R^+ is a real vector space. If not state the axioms that fail. *11 K3 CO3*

OR

b) $P_3(R)$ is a vector space of polynomials of degree less than or equal to 3 over R. Test whether $x^3 - 3x + 5$ is a linear combination of $x^3 + 2x^2 - x + 1$ and $x^3 + 3x^2 - 1$. *11 K3 CO3*

26. a) Verify that the set $\{u_1, u_2, u_3\}$ where $u_1 = (0,1,1), u_2 = (1,3,1), u_3 = (3,2,1)$ in R^3 is a basis over R. Construct an orthonormal basis by Gram – Schmidt method. *11 K3 CO4*

OR

b) Construct the QR decomposition for the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. *11 K3 CO4*

27.	a)	If T is a linear operator on R^3 defined by by $T(a,b,c) = (7a - 4b + 10c, 4a - 3b + 8c, -2a + b - 2c)$ and an ordered basis of R^3.Compute the eigen values and eigen vectors of T.	11	K3	CO5
		OR			
	b)	Let $T: R^3 \rightarrow R^3$ be defined by $T(x,y,z) = (x + y, x - y, 2x + z)$. Compute N(T), R(T) and hence verify the dimension theorem.	11	K3	CO5
28.	a)	Construct the Singular value decomposition for the matrix $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$.	11	K3	CO6
		OR			
	b)	What is Principal Component Analysis? Discuss the applications of Principal Component Analysis in Image Processing and Machine Learning.	11	K3	CO6