Reg. No.																

Question Paper Code	13889

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025

Third Semester

Computer and Science Engineering (Cyber Security)

24BSMA306 - LINEAR ALGEBRA AND NUMBER THEORY

Regulation - 2024

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

		Marks	K – Level	CO
1.	The zero vector in a vector space satisfies (a) $v + 0 = v$ (b) $0 + v = v$ (c) $v + v = 0$ (d) $0 = 1$	1	K1	CO1
2.	A basis of a vector space is (a) any subset of vectors (b) a set of linearly dependent vectors (c) a linearly independent set that spans the space (d) the set containing only zero vector	1	K1	CO1
3.	If a matrix has distinct Eigen values, it is always (a) Symmetric (b) Singular (c) Diagonalizable (d) skew -symmetric	1	K1	CO2
4.	The null space of a linear transformation is (a) The set of all vectors mapped to zero (b) The set of all nonzero vectors (c) The set of all unit vectors (d) The set of all basis vectors	1	K1	CO2
5.	The inner product of two orthogonal vectors is (a) 1 (b) -1 (c) 0 (d) Undefined	1	K1	CO3
6.	The adjoint of an operator is defined with respect to (a) The norm of the vector space (b) The basis of the space (c) The inner product (d) The determinant of the matrix	1	K1	CO3
7.	if a divides b then $b = aq$ for some (a) Prime number q (b) Integer q (c) Real number q (d) Rational number q	1	K1	CO4
8.	A prime number has exactly (a) One positive divisor (b) Two positive divisors (c) Three positive divisors (d) Infinitely many divisors	1	K1	CO4
9.	The number of solutions of $2x \equiv 4 \ (mod\ 6)$ is (a) 1 (b) 2 (c) 3 (d) 6	1	K2	CO5
10.	The value of $\emptyset(9)$ is (a) 3 (b) 6 (c) 8 (d) 9	1	K2	CO6

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

11.	Check whether the set of vectors in $R^3(R)$ given by $\{(1,2,1),(2,1,0),\ (1,-1,2)\}$ are linearly dependent or linearly independent.	2	K2	CO1
12.	Check in $V_3(R)$ the vector (-2, 0, 3) can be expressed as a linear combination of vectors (1, 3, 0) and (2, 4, -1).	2	K2	CO1
13.	Find the linear transformation $T: P_2(R) \longrightarrow P_3(R)$ given by $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$ with respect to the standard basis.	2	K2	CO2
14.	Find the Eigenvalues of the matrix $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$.	2	K2	CO2
15.	If $\alpha = (1,2,3,4)$ and $\beta = (2,0,-3,1)$ then find $\|\alpha + \beta\|$.	2	K2	CO3

16. Find the Minimal solution to the system $x + 2y - z = 12$. *2 K2 CO3*

17. Express two 10110_{two} in base ten. *2 K2 CO4*

18. Using Euclidean Algorithm, find $(4076,1024)$. *2 K2 CO4*

19. When does the linear congruence ax ≡ b (mod m) has a unique solution? *2 K1 CO5*

20. Recall the Chinese Remainder Theorem. *2 K1 CO5*

21. State the Wilson's theorem. *2 K1 CO6*

22. Find the value of $\phi(28)$. *2 K2 CO6*

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) Determine whether the following sets are subspace of $V_3(R)$ under the operations of addition and scalar multiplication defined on $V_3(R)$. *11 K3 CO1*
$\{(a_1, a_2, a_3)/a_1 + a_2 = 0\}$,$\{(a_1, a_2, a_3)/a_1 + a_2 + 2a_3 = 0\}$.

OR

b) In $V_3(R)$ where R is the field of real numbers, examine each of the following sets of vectors for linear dependence *11 K3 CO1*
(i) $S = \{(1, 2, 0), (0, 3, 1), (-1, 0, 1)\}$
(ii) $S = \{(1, 3, 2), (1, -7, -8), (2, 1, -1)\}$

24. a) Test the matrix $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ for Diagonalizablity. If A is diagonalizable, find an invertible matrix Q and a diagonal matrix D such that $A^{-1}AQ = D$. Also, using this result, compute A^n for any positive integer n. *11 K3 CO2*

OR

b) Let $T: V_3(R) \longrightarrow V_3(R)$ be the linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find the basis and dimension of $R(T)$ and $N(T)$. *11 K3 CO2*

25. a) Apply Gram Schmidt process, to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where *11 K3 CO3*
$v_1 = (1, 0, 1)$, $v_2 = (1,3,1)$ and $v_3 = (3\ 2, 1\ )$.

OR

b) For the data {(1,2),(2,3),(3,5),(4,7)}, use least squares approximation method to find the best fit of linear equation. *11 K3 CO3*

26. a) Let a be any integer and b be a positive integer. Then prove that there exist unique integers q and r such that $a = bq + r$ where $0 \leq r \leq b$. *11 K3 CO4*

OR

b) State and Prove Fundamental Theorem of Arithmetic. *11 K3 CO4*

27. a) Solve the Linear Diophantine Equation: $1076x + 2076y = 3076$. *11 K3 CO5*

OR

b) Find an integer that has a remainder of 3 when divided by 7 and 13, but is not divisible by 12. *11 K3 CO5*

28. a) State and Prove Fermat's little theorem. *11 K3 CO6*

OR

b) Using Euler's theorem, find the remainder when 245^{1040} is divided by 18. *11 K3 CO6*