| Reg. No. | | | | | | | | | | | | | | | |
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| Question Paper Code | 14171 |
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**M.E. / M.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025**

First Semester

**Computer Science and Engineering**

(Common to Computer Science and Engineering (Specialization in Networking)

**24PCSMA104 - APPLIED PROBABILITY AND STATISTICS**

Regulations - 2024

Duration: 3 Hours | Max. Marks: 100

**PART - A (MCQ) (10 × 1 = 10 Marks)**

Answer ALL Questions

| | | Marks | K – Level | CO |
|---|---|---|---|---|
| 1. | The parameter of Binomial distribution is<br>(a) $n$ (b) $p$ (c) $n, p$ (d) $q$ | 1 | K1 | CO1 |
| 2. | The mean of uniform distribution is<br>(a) $a+b$ (b) $a-b$ (c) $\frac{a-b}{2}$ (d) $\frac{a+b}{2}$ | 1 | K1 | CO1 |
| 3. | If $(X,Y)$ are two-dimensional continuous independent random variable then<br>(a) $f(x,y)=fX(x)+fY(y)$ (b) $f(x,y)=fX(x)fY(y)$ (c) $f(x\|y)=fY(y)$ (d) $f(y\|x)=fX(x)$ | 1 | K1 | CO2 |
| 4. | The relation between correlation coefficient and regression coefficients is<br>(a) $r=\sqrt{b_{yx}b_{xy}}$ (b) $r=\sqrt{b_{yy}b_{xy}}$ (c) $r=\sqrt{b_{xx}b_{xy}}$ (d) $r=\sqrt{b_{yy}b_{xx}}$ | 1 | K1 | CO2 |
| 5. | Let $\theta$ be an unknown parameter and $T_1$ be an unbiased estimator of $\theta$. If $Var(T_1) \leq Var(T_2)$ for $T_2$ to be any other unbiased estimator, then $T_1$ is known as<br>(a) minimum variance unbiased estimator<br>(b) Unbiased and efficient estimator.<br>(c) Consistent and efficient estimator.<br>(d) Unbiased, consistent and minimum variance estimator. | 1 | K1 | CO3 |
| 6. | An estimator $T_n$ is said to be an unbiased estimator of $\chi(\theta)$, for all $\theta \epsilon \boldsymbol{\Theta}$ if<br>(a) $E(T_n)=\chi(\theta)$ (b) $E(T_n)>\chi(\theta)$ (c) $E(T_n)<\chi(\theta)$ (d) $E(T_n)\neq\chi(\theta)$ | 1 | K1 | CO3 |
| 7. | Chi Square is used to<br>(a) test the significant differences of two mean<br>(b) test the significant differences of two variances<br>(c) test the goodness of fit<br>(d) None of the above. | 1 | K1 | CO4 |
| 8. | Which of the following test is used to test the significance of differences between variances of two populations?<br>(a) t – test (b) F- test (c) Z – test (d) Chi Square test | 1 | K1 | CO4 |
| 9. | The proportion of total population variance due to k$^{th}$ principal component is<br>(a) $\frac{\lambda_k}{\lambda_1+\lambda_2+\cdots+\lambda_p}$ (b) $\frac{\lambda_1}{\lambda_1+\lambda_2+\cdots+\lambda_p}$ (c) $\frac{\lambda_1+\lambda_2+\cdots+\lambda_p}{\lambda_k}$ (d) $\frac{\lambda_k}{\lambda_p}$ | 1 | K2 | CO5 |
| 10. | A matrix is said to be positive definite if<br>(a) $(X'AX = 0 \; \forall \; x \neq 0$ (b) $X'AX < 0 \; \forall \; x \neq 0$<br>(c) $X'AX > 0 \; \forall \; x \neq 0$ (d) $AX' < 0 \; \forall \; x \neq 0$ | 1 | K1 | CO5 |

**PART - B (12 × 2 = 24 Marks)**

Answer ALL Questions

| | | | | |
|---|---|---|---|---|
| 11. | A discrete random variable $X$ has moment generating function $M_X(t) = \left(\frac{1}{4}+\frac{3}{4}e^t\right)^5$. Find $E(X)$. | 2 | K3 | CO1 |
| 12. | State memoryless property of geometric distribution. | 2 | K1 | CO1 |
| 13. | If $y = 2x - 3$ and $y = 5x + 7$ are the two regression lines, find the mean values of $x$ and $y$. | 2 | K2 | CO2 |

14. The coefficient of correlation between two variables $X$ and $Y$ is 0.48, covariance is 36 and the variance of $X$ is 16. Find the standard deviation of $Y$. *2 K2 CO2*

15. If $T$ is an unbiased estimator for $\theta$, show that $T^2$ is a biased estimator for $\theta^2$. *2 K2 CO3*

16. Define likelihood function. *2 K1 CO3*

17. Define level of significance. *2 K1 CO4*

18. Ten oil tins are taken from an automatic filling machine. The mean weight of the tins are15.8 kg and standard deviation is 0.50 kg. Does the sample mean differ significantly from the intended weight 16 kg? *2 K2 CO4*

19. If $X = \begin{pmatrix} 42 & 4 \\ 52 & 5 \\ 48 & 4 \end{pmatrix}$ then find $\bar{X}$. *2 K2 CO5*

20. If $\sum = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$, then find the standard deviation matrix $V^{1/2}$. *2 K2 CO5*

21. Let $X$ be a uniformly distributed random variable with mean 1 and variance $\frac{4}{3}$. Find ($X$<0). *2 K2 CO1*

22. If $X$, $Y$ are independent random variables with Var(X) = 2, Var(X) = 3, then find the value of Var(3X+2Y). *2 K2 CO2*

**PART - C (6 × 11 = 66 Marks)**

Answer ALL Questions

23. a) A random variable X has the following probability function: *11 K3 CO1*

| Value of $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

Find $k$

Evaluate $P(X < 6), P(X \geq 6)$ and $P(0 < X < 5)$

Find the minimum value of a such that $P(X \leq a) > \frac{1}{2}$

Determine the distribution function of $X$.

**OR**

b) A continuous random variable $X$ has the probability density function$f(x) = kx^2e^x$, $x$≥0. Find the value of $k$, $r^{th}$ moment about origin also find mean and variance. *11 K3 CO1*

24. a) Find the coefficient of correlation between X and Y from the data given below. *11 K3 CO2*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $X$: | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| $Y$: | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

**OR**

b) The joint probability density function of random variables $X$ and $Y$ is given by $f(x,y) = kxye^{-(x^2+y^2)}$,$x$>0,>0. Find the value of k and also prove that $X$ and $Y$ are independent. *11 K3 CO2*

25. a) Let $X_1,X_2....X_n$ be a random sample size n from the Poisson distribution $f(x/\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$ *where* $0 \leq \lambda \leq \infty$. Obtain the maximum likelihood estimator of $\lambda$. *11 K3 CO3*

**OR**

b) A random sample $(X_1, X_2, X_3, X_4, X_5)$of size 5 is drawn from a normal population with unknown mean $\mu$. Consider the following estimators to estimate $\mu$ *11 K3 CO3*

i) $t_1 = \frac{X_1+X_2+X_3+X_4+X_5}{5}$ ii) $t_2 = \frac{X_1+X_2}{2} + X_3$ iii) $t_3 = \frac{2X_1+X_2+\lambda X_3}{3}$

where $\lambda$ is such that $t_3$ is an unbiased estimator of $\mu$. Find $\lambda$. Are $t_1$ and $t_2$ unbiased? State giving reasons, the estimator which the best among $t_1, t_2$ *and* $t_3$

26. a) The theory predicts that the proportion of beans in four given group should be $9:3:3:1$. In an examination with 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? *11 K3 CO4*

**OR**

b) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. *11 K3 CO4*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sample I | 18 | 13 | 12 | 15 | 12 | 14 | 16 | 14 | 15 |
| Sample II | 16 | 19 | 13 | 16 | 18 | 13 | 15 | | |

Do the estimates of the population variance differ significantly at 5% level?

27. a) Analyse the covariance matrix of a 3-dimensional vector $X = (X_1, X_2, X_3)$ is given by $\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$. Determine the correlation matrix and correlation between $X_1$ and $\frac{X_2}{2} + \frac{X_3}{2}$. *11 K4 CO5*

**OR**

b) Analyse the covariance matrix $\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and perform a principal component analysis (PCA).Compute the eigen values and eigenvectors, construct the principal components $Y_1, Y_2, Y_3$ and explain how the covariance relationships among $X_1$, $X_2$ and $X_3$ influence the orientation of the principal directions. *11 K4 CO5*

28. a) (i) Find Moment Generating function of Binomial distribution. Hence find mean and variance. *5 K3 CO1*

(ii) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same at 5 % level. *6 K3 CO4*

**OR**

b) (i) The number of accidents in a year attributed to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find number of drivers with (a) no accident in a year, (b) more than 3 accidents in a year. *5 K3 CO1*

(ii) The mean height of two samples of 1000 and 2000 members are respectively 67.5 and 68 inches. Can they be regarded as drawn from the same population with standard deviation 2.5 inches? *6 K3 CO4*