



13. Find the extremal of the functional  $\int_{x_0}^{x_1} (y'^2 - y^2) dx$ . 2 K2 CO2
14. Define isoperimetric problem. 2 K1 CO2
15. A continuous random variable  $X$  has probability density function  $f(x) = kx^2, 0 \leq x \leq 1$ . Find the value of  $k$ . 2 K2 CO3
16. The probability that a candidate can pass in an examination is 0.6. What is the probability that he will pass in third trial? 2 K2 CO3
17. Derive the moment generating function of a random variable which follows uniform distribution. 2 K2 CO4
18. Use the graphical method to solve the following LPP:  $Max Z = 3x + 4y$  s.t  $x - y \leq -1, -x + y \leq 0, x, y \geq 0$ . 2 K2 CO4
19. Explain how maximization problems are solved in the assignment model. 2 K1 CO4
20. Find the Fourier series of  $f(x) = x, -\pi < x < \pi$ . 2 K2 CO5
21. State Parseval's theorem. 2 K1 CO5
22. Define generalized Fourier series. 2 K1 CO5

**PART - C (6 × 11 = 66 Marks)**

Answer ALL Questions

23. a) Construct the QR-decomposition of the matrix  $A = \begin{pmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{pmatrix}$ . 11 K3 CO1

**OR**

- b) Write the least square solutions to  $x + 2y + z = 1, 3x - y = 2, 2x + y - z = 2$  and  $x + 2y + 2z = 1$ . 11 K3 CO1

24. a) State and prove Brachistochrone problem. 11 K3 CO2

**OR**

- b) Using Ritz method, solve the boundary value problem  $y'' - y + x = 0, 0 \leq x \leq 1, y(0) = y(1) = 0$ . 11 K3 CO2

25. a) (i) A factory has two machines. Past records show that the first machine produces 40 percent of output and the second machine produces 60 percent of output. Further, 4 percent and 2 percent of products produced by the first machine and the second machine were defectives. If a defective item is drawn at random, what is the probability that the defective item was produced by the first machine or second machine? 6 K3 CO3

- (ii) The pmf of a random variable  $X$  is defined as  $P(X = 0) = 3c^2, P(X = 1) = 4c - 10c^2, P(X = 2) = 5c - 1$  where  $c > 0$  and  $P(X = r) = 0, r \neq 0, 1, 2$ . Find 1.  $c$  2. Cdf 3. The largest value of  $x$  for which  $F(x) < 1/2$ . 4.  $P(0 < X < 2/X > 0)$ . 5 K3 CO3

**OR**

- b) (i) Given  $F(x) = 1 - \frac{4}{x^2}, x > 2$ . Find 1.  $P(X < 3)$  2.  $P(4 < X < 5)$  3.  $P(X \geq 3)$  using both cdf and pdf 6 K3 CO3

- (ii) Given a random variable  $X$  with pdf  $f(x) = kx(2 - x), 0 \leq x \leq 2$ . Find the  $r^{\text{th}}$  moment about origin and hence find mean and variance. 5 K3 CO3

26. a) Solve the LPP using Big M method: 11 K3 CO4  
 Maximise  $Z = 2x_1 + x_2 + x_3$   
 subject to  $4x_1 + 6x_2 + 3x_3 \leq 8, 3x_1 - 6x_2 - 4x_3 \leq 1, 2x_1 + 3x_2 - 5x_3 \geq 4,$   
 $x_1, x_2, x_3 \geq 0$ .

**OR**

- b) Solve the following transportation problem: A company operates three coal mines  $A, B$  and  $C$  which provide 400, 500, and 700 ton respectively per week. Orders for 500, 400, 300, 300 and 600 ton per week have been received from customers  $C_1, C_2, C_3, C_4$  and  $C_5$  respectively. Transportation cost in Rs. per tonne from each mine to each customer are given below: 11 K3 CO4

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$A$	4	16	1	16	14	400
$B$	18	10	8	12	12	500
$C$	6	1	4	13	2	700
Demand	500	400	300	300	600	

Find the weekly shipping schedule which minimizes the total expenses.

27. a) Find the eigen values and eigen functions of  $y'' + \lambda y = 0; 0 < x < p, y(0) = y(p) = 0$ . 11 K3 CO5

**OR**

- b) Find the Fourier series for  $f(x) = x^2, -\pi < x < \pi$ . Hence deduce that  $\frac{1}{1^4} + \frac{1}{2^4} + \dots = \frac{\pi^4}{90}$ . 11 K3 CO5

28. a) (i) The number of accidents in a year to taxi drivers in city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with no accidents in a year. Also find the number of drivers with more than 3 accidents in a year. 6 K3 CO3

- (ii) The daily consumption of milk in excess of 20,000 gallons is approximately exponentially distributed with parameter  $\lambda = \frac{1}{3000}$ . The city has a daily stock of 35,000 gallons. What is the probability that of two days selected at random, the stock is insufficient for both the days? 5 K3 CO3

**OR**

- b) State and prove memoryless property of exponential distribution. Use it to solve the problem: 11 K3 CO3

The time in hours required to repair a machine is exponentially distributed with parameter  $\frac{1}{2}$ .

What is the probability that the repair time exceeds 2 hours?

What is the probability that the repair time takes at least 10 hours given that its duration exceeds 9 hours.