Reg. No.																

Question Paper Code	13694

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2025

First Semester

Computer Science and Business Systems

20BSMA102 - DISCRETE MATHEMATICS

Regulations - 2020

Duration: 3 Hours Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

		Marks	K – Level	CO
1.	What is the negation of the statement "Sam is rich and happy"? (a) Sam is poor or unhappy (b) Either Sam is poor or happy (c) Either Sam is poor or unhappy (d) Sam is not rich and happy.	1	K1	CO1
2.	$p \to q$ is logically equivalent to________ (a) $\neg p \lor \neg q$ (b) $p \lor \neg q$ (c) $\neg p \lor q$ (d) $\neg p \land q$	1	K1	CO1
3.	If m pigeons are assigned to n pigeonholes, then there must be a pigeonhole containing at least (a) $\left[\left(\frac{m-1}{n}\right)\right]+1$ pigeons (b) $\left[\left(\frac{m-1}{n}\right)\right]-1$ pigeons (c) $\left[\left(\frac{m+1}{n}\right)\right]+1$ pigeons (d) $\left[\left(\frac{m-n}{n}\right)\right]+1$ pigeons	1	K1	CO2
4.	If 9 colours are used to paint 100 houses then at least how many houses will be of the same colour (a) 15 (b) 13 (c) 12 (d) 14	1	K2	CO2
5.	How many rows are there in the truth table for a function with 3 variables? (a) 4 (b) 6 (c) 8 (d) 12	1	K2	CO3
6.	The dual of $A+0=A$ is (a) $A \cdot 1 = A$ (b) $A+1=A$ (c) $A \cdot 0 = A$ (d) $A+0=1$	1	K1	CO3
7.	A Hamiltonian path in a graph (a) Visits every edge exactly once (b) Visits every vertex exactly once (c) Visits every vertex and edge exactly once (d) Must be a cycle	1	K1	CO4
8.	The number of edges in a tree with n vertices is (a) $n-1$ (b) n (c) $n+1$ (d) $2n$	1	K1	CO4
9.	Let $G=\{1,-1.i,-i\}$ is group under multiplication then how many elements are self-invertible in G? (a) 1 (b) 2 (c) 3 (d) 4	1	K1	CO5
10.	If H_1 and H_2 are subgroups of the group G, Then, which one of the following is also a subgroup of G? (a) $H_1 \cup H_2$ (b) $H_1 \cap H_2$ (c) $H_1 \Delta H_2$ (d) none of these	1	K1	CO5

PART - B (12× 2 = 24 Marks)

Answer ALL Questions

11.	Prove that $\left(p \land (p \to q)\right) \to q \Rightarrow T$.	2	K2	CO1
12.	Use indirect method to prove that the conclusion $(\exists x)Q(z)$ from $(\forall x)\left(P(x) \to Q(x)\right)$ and $(\exists y)P(y)$.	2	K2	CO1
13.	Find the dual of the statement $(p \land q) \to (p \to q)$.	2	K2	CO1
14.	Solve the recurrence relation $a_n - 2a_{n-1} = 3^n$, $a_1 = 5$.	2	K2	CO2
15.	In any group of 100 people, several will have birthdays in the same month. At least how many must have birth days in the same month.	2	K2	CO2

16. Prove that if n is an integer and n is odd, then n^2 is odd. *2 K2 CO2*

17. Prove that the zero element of a Boolean algebra is unique. *2 K2 CO3*

18. Simplify the Boolean expression $xy' + z + (x' + y)z'$. *2 K2 CO3*

19. State and prove the hand shaking theorem. *2 K2 CO4*

20. Define a tree with an example. *2 K1 CO4*

21. Prove that the identity element of a group is unique. *2 K2 CO5*

22. Show that the set {1, 2, 3, 4 } is a group under multiplication modulo 5. *2 K2 CO5*

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) Obtain the principal conjunctive normal form (PCNF) and principal disjunctive normal form (PDNF) of $(\neg p \rightarrow q) \wedge (q \leftrightarrow p)$. *11 K3 CO1*

OR

b) Verify the validity of the following argument: *11 K3 CO1*
"If A works hard, then B or C will enjoy themselves. If B enjoys himself, then A will not work hard. If D enjoys himself, then C will not. Therefore, if A works hard, then D will not enjoy himself."

24. a) Prove by using Mathematical induction *11 K3 CO2*

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

OR

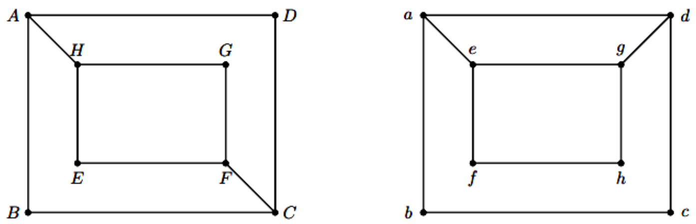
b) Solve the recurrence relation $a_{n+1} - a_n = 2n + 3, n \geq 0, a_0 = 1$. *11 K3 CO2*

25. a) In a Boolean algebra, prove that the following statements are equivalent. *11 K3 CO3*
$i)\ a + b = b \quad ii)\ a.b = a \quad iii)\ a' + b = 1 \quad iv)\ a.b' = 0$.

OR

b) Minimize the function $f(a,b,c,d) = \Sigma(1,2,4,5,6,11,12,13,14,15)$ using Karnaugh map method. *11 K3 CO3*

26. a) Examine whether the following pairs of graphs G_1 and G_2 given in figures are isomorphic or not. *11 K3 CO4*



G_1 G_2

OR

b) Prove that the maximum number of edges in a simple disconnected group G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. *11 K3 CO4*

27. a) Show that the intersection of any two subgroup of a group G is also a subgroup of G. *11 K3 CO5*

OR

b) State and prove Lagrange's theorem. *11 K3 CO5*

28. a) (i) Prove that $1^2 + 2^2 + \ ... \ + n^2 = \frac{n(n+1)(2n+1)}{6}$, by mathematical induction. *6 K3 CO2*

(ii) In any Boolean algebra, Show that *5 K3 CO3*

$(a + b')\ (b + c')\ (c + a') = (a' + b)\ (b' + c)\ (c' + a)$.

OR

b) (i) A man hiked for 10 hours and covered a total distance of 45 km. It is known that he hiked 6 km in the first hour and only 3 km in the last hour. Show that he must have hiked at least 9 km within a certain period of 2 consecutive hours. *6 K3 CO2*

(ii) If $a, b \in S$, $S = \{1,2,4,8\}$ and $a + b = LCM(a, b)$, $a.b = GCD(a, b)$ and $a' = 8/a$, show that $\{S, +, . \ , ', 1 , 8\}$ is not a Boolean algebra. *5 K3 CO3*