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Question Paper Code	13754
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**B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2025**

Second Semester

**Computer Science and Business Systems**

**20BSMA202 - LINEAR ALGEBRA**

Regulations - 2020

Duration: 3 Hours

Max. Marks: 100

**PART - A (MCQ) (10 × 1 = 10 Marks)**

Answer ALL Questions

- |                                                                                                                   | Marks | K- | CO  |
|-------------------------------------------------------------------------------------------------------------------|-------|----|-----|
|                                                                                                                   | Level |    |     |
| 1. For Cramer's Rule to be applicable, the determinant of the coefficient matrix must be:                         | 1     | K1 | CO1 |
| (a) Non-zero      (b) Zero      (c) Equal to one      (d) Greater than or equal to zero                           |       |    |     |
| 2. If a matrix A is invertible, then which of the following is true?                                              | 1     | K1 | CO1 |
| (a) Inverse of A does not exist.      (b) The determinant of A is zero.                                           |       |    |     |
| (c) The system Ax=B has a unique solution.      (d) The rows of A are linearly dependent.                         |       |    |     |
| 3. The rank of the matrix $A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$                                     | 1     | K2 | CO2 |
| (a) 1      (b) 2      (c) 3      (d) 4                                                                            |       |    |     |
| 4. Given vectors $u = (1,1)$ and $v = (2,3)$ , which of the following vectors is a linear combination of u and v? | 1     | K2 | CO2 |
| (a) (3,4)      (b) (4,5)      (c) (5,6)      (d) (1,1)                                                            |       |    |     |
| 5. What is the dimension of the vector space $\mathbb{R}^3$ ?                                                     | 1     | K1 | CO3 |
| (a) 1      (b) 2      (c) 3      (d) 4                                                                            |       |    |     |
| 6. The norm of the vector $(2, -3, 4)$ is                                                                         | 1     | K2 | CO3 |
| (a) $\sqrt{29}$ (b) 29      (c) 841      (d) 49                                                                   |       |    |     |
| 7. A matrix A is a Hermitian matrix , if                                                                          | 1     | K1 | CO4 |
| (a) $A = A^T$ (b) $A = \bar{A}^T$ (c) $A = \bar{A}^T$ (d) none of the above                                       |       |    |     |
| 8. Which of the following is an example of a linear transformation?                                               | 1     | K2 | CO4 |
| (a) $T(x, y) = (x + y, y - x)$ (b) $T(x, y) = (x^2, 4y)$                                                          |       |    |     |
| (c) $T(x, y) = (x, \ln(y))$ (d) $T(x, y) = (xy, x + y)$                                                           |       |    |     |
| 9. Principal component analysis can be used for projecting and visualizing data in lower dimensions.              | 1     | K1 | CO5 |
| (a) True      (b) False      (c) Partially true      (d) None of the above                                        |       |    |     |
| 10. In SVD of a matrix A, the eigenvalues are calculated for                                                      | 1     | K1 | CO5 |
| (a) A matrix      (b) $A^T$ matrix      (c) $AA^T$ matrix      (d) None of the above                              |       |    |     |

**PART - B (12 × 2 = 24 Marks)**

Answer ALL Questions

- |                                                                                                                                                                              | Marks | K- | CO  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----|-----|
|                                                                                                                                                                              | Level |    |     |
| 11. What is the inverse of the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ ?                                                                                   | 2     | K2 | CO1 |
| 12. Solve the following system of equations $3x + 4y = 2$ and $5x + 6y = 4$ .                                                                                                | 2     | K2 | CO1 |
| 13. If $A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}$ , find B such that $AB = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 6 & 1 \\ 5 & 6 & 4 \end{pmatrix}$ . | 2     | K2 | CO1 |
| 14. Define rank of a matrix.                                                                                                                                                 | 2     | K1 | CO2 |
| 15. Determine whether the vectors $(1, -2, 3), (5, 6, -1), (3, 2, 1)$ form a linearly dependent or independent set in $\mathbb{R}^3$ .                                       | 2     | K2 | CO2 |
| 16. For what value of k, the system of equations $x + 2y = 5, 3x + ky - 15 = 0$ has no solution.                                                                             | 2     | K1 | CO2 |
| 17. Define a subspace of a Vector space.                                                                                                                                     | 2     | K2 | CO3 |
| 18. Write down the basis for $M_2(\mathbb{R})$ .                                                                                                                             | 2     | K2 | CO3 |

- |                                                     |   |               |
|-----------------------------------------------------|---|---------------|
| 19. Define a Linear Transformation.                 | 2 | <i>K1 CO4</i> |
| 20. Define a Positive definite matrix.              | 2 | <i>K1 CO4</i> |
| 21. State Singular value decomposition theorem.     | 2 | <i>K1 CO5</i> |
| 22. State any two applications of Image Processing. | 2 | <i>K1 CO5</i> |

**PART - C (6 × 11 = 66 Marks)**

Answer ALL Questions

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|--------|----|---------------|
| 23. a) | 11 | <i>K3 CO1</i> |
|--------|----|---------------|

Solve the equations  $3x + y + 2z = 3$ ;  $2x - 3y - z = -3$ ;  $x + 2y + z = 4$  using Cramer's Rule.

**OR**

- |    |    |               |
|----|----|---------------|
| b) | 11 | <i>K3 CO1</i> |
|----|----|---------------|
- By finding  $A^{-1}$ , solve the linear equation  $AX=B$  where  $A = \begin{pmatrix} 3 & 4 & 5 \\ 1 & 2 & 0 \\ 5 & 1 & 1 \end{pmatrix}$ ,
- $$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}.$$

- |        |    |               |
|--------|----|---------------|
| 24. a) | 11 | <i>K3 CO2</i> |
|--------|----|---------------|

Calculate the rank of the matrix  $\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix}$ .

**OR**

- |    |    |               |
|----|----|---------------|
| b) | 11 | <i>K3 CO2</i> |
|----|----|---------------|
- Show that the equations  $5x + 3y + 7z = 4$ ,  $3x + 26y + 2z = 9$ ,  $7x + 2y + 10z = 5$  are consistent and solve them by rank method.

- |        |    |               |
|--------|----|---------------|
| 25. a) | 11 | <i>K3 CO3</i> |
|--------|----|---------------|
- Let  $S = \{v_1, v_2, v_3\}$  where  $v_1 = (1, 2, -3)$ ,  $v_2 = (2, 5, 1)$ ,  $v_3 = (-1, 1, 4)$ . Verify whether  $S$  forms a basis or not.

**OR**

- |    |    |               |
|----|----|---------------|
| b) | 11 | <i>K3 CO3</i> |
|----|----|---------------|
- Let  $R^3$  have the Euclidean inner product. Use Gram – Schmidt process to construct the basis  $\{u_1, u_2, u_3\}$  into an orthonormal basis, where  $u_1 = (1, 1, 1)$ ,  $u_2 = (0, 1, 1)$ ,  $u_3 = (0, 0, 1)$ .

- |        |    |               |
|--------|----|---------------|
| 26. a) | 11 | <i>K3 CO4</i> |
|--------|----|---------------|
- Find the matrix of T in the standard basis for the transformation  $T : P_2(R) \rightarrow P_2(R)$  where  $T(f(x)) = f'(x)$ .

**OR**

- |    |    |               |
|----|----|---------------|
| b) | 11 | <i>K3 CO4</i> |
|----|----|---------------|
- Prove that there exist a linear transformation  $T : R^2 \rightarrow R^3$  such that  $T(1, 1) = (1, 0, 2)$ ,  $T(2, 3) = (1, -1, 4)$ . Find  $T(8, 11)$ .

- |        |    |               |
|--------|----|---------------|
| 27. a) | 11 | <i>K3 CO5</i> |
|--------|----|---------------|
- Determine the singular value decomposition of  $A = \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}$ .

**OR**

- |    |    |               |
|----|----|---------------|
| b) | 11 | <i>K3 CO5</i> |
|----|----|---------------|
- Apply Principal Component Analysis (PCA) to reduce the data to a 1-dimensional space.

4	5	6	7	8
6	7	8	9	10

- |           |   |               |
|-----------|---|---------------|
| 28. a (i) | 6 | <i>K3 CO4</i> |
|-----------|---|---------------|
- Let  $T : R^3 \rightarrow R^2$  be defined by  $T(x, y, z) = (2x - y, 3z)$ . Verify whether  $T$  is linear or not. Find  $N(T)$ .

- |      |   |               |
|------|---|---------------|
| (ii) | 5 | <i>K3 CO5</i> |
|------|---|---------------|
- Explain Applications of matrices in Machine Learning.

**OR**

- |       |   |               |
|-------|---|---------------|
| b (i) | 6 | <i>K3 CO4</i> |
|-------|---|---------------|
- Let  $T : R^2 \rightarrow R^3$  by  $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$ . Find nullity of  $T$  and rank of  $T$ .

- |      |   |               |
|------|---|---------------|
| (ii) | 5 | <i>K3 CO5</i> |
|------|---|---------------|
- Discuss the applications of matrices with Principal Component Analysis in Image Processing.

