Reg. No.															

Question Paper Code	13708

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2025

Third Semester

Mechanical Engineering

(Common to Mechanical and Automation Engineering)

20BSMA303 - PARTIAL DIFFERENTIAL EQUATIONS AND PROBABILITY THEORY

Regulations - 2020

(Use of Statistical Tables is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
1.	Elimination of arbitrary function f from $z = f(x^2 - y^2)$ will result in the partial differential equation: (a) $px + qy = 0$ (b) $qx + py = 0$ (c) $qx - py = 0$ (d) $px - qy = 0$	1	K3	CO1
2.	The complete solution of the first order partial differential equation $py = qx$ is (a) $z = \frac{ax^2}{2} + ay + c$ (b) $z = ax + \frac{ay^2}{2} + c$ (c) $z = \frac{ax^2}{2} + \frac{ay^2}{2} + c$ (d) $z = ax + ay + c$	1	K3	CO1
3.	The second order partial differential equation $x^2 f_{xx} + (1 - y^2) f_{yy} = 0,\ x \neq 0,$ $-1 < y < 1$ is (a) elliptic (b) parabolic (c) hyperbolic (d) quadratic	1	K2	CO2
4.	α^2in one-dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ stand for (a) thermal conductivity (b) diffusivity (c) specific heat (d) density	1	K1	CO2
5.	If $F\{f(x)\} = F(s)$ then $F\{f(x) cos ax\} =$ (a) $\frac{1}{2}[F(s+a) + F(s-a)]$ (b) $\frac{1}{2}[F(s+a) - F(s-a)]$ (c) $\frac{1}{2}[F(a+s) - F(a-s)]$ (d) $\frac{1}{2}[F(a+s) + F(a-s)]$	1	K3	CO3
6.	Which of the following functions is self-reciprocal under both Fourier sine and cosine transforms? (a) $\sqrt{x}$ (b) $\sqrt{\frac{1}{x}}$ (c) $e^{-\frac{x^2}{2}}$ (d) $e^{\frac{x^2}{2}}$	1	K2	CO3
7.	Bag 1 contains 2 red and 1 black balls and bag II contains 3 red and 2 black balls. What is the probability that a ball drawn from one of the bags is red? (a) $\frac{17}{30}$ (b) $\frac{19}{30}$ (c) $\frac{1}{2}$ (d) $\frac{13}{30}$	1	K3	CO4
8.	The moment generating function of the standard normal variate Z is (a) $e^{-\frac{t^2}{2}}$ (b) $e^{\frac{t^2}{2}}$ (c) e^{-t^2} (d) e^{t^2}	1	K2	CO4
9.	If (X, Y) is a two-dimensional continuous random variable then X and Y are independent if (a) $f(x,y) = f_X(x) + f_Y(y)$ (b) $f(x,y) = f_X(x) f_Y(y)$ (c) $f(x\|y) = f_Y(y)$ (d) $f(y\|x) = f_X(x)$	1	K2	CO5
10.	If $f(x,y) = k(1 - x - y),\ 0 < x, y < \frac{1}{2}$ is a joint density function, then the value of k is (a) $\frac{1}{2}$ (b) 4 (c) 8 (d) $\frac{1}{8}$	1	K3	CO5

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

11.	Form a partial differential equation by eliminating the arbitrary constants a, b from $z = ax^2 + by^2$.	2	K2	CO1
12.	Solve $p = 2qx$.	2	K2	CO1
13.	Find the particular integral $(D^2 - 2DD' + D'^2)z = e^{x-y}$.	2	K2	CO1
14.	Classify the partial differential equation $f_{xx} - 2f_{xy} = 0 \; for \; x > 0 \; \& \; y > 0$.	2	K2	CO2
15.	Write all the possible solutions of one-dimensional heat equation.	2	K1	CO2
16.	Define the terms: (1) temperature change (2) temperature gradient.	2	K1	CO2
17.	Write down the complex Fourier transform pair.	2	K1	CO3
18.	State the convolution theorem on Fourier transform.	2	K1	CO3
19.	State Bayes' theorem on conditional probability.	2	K1	CO4
20.	State the memoryless property of geometric distribution.	2	K1	CO4
21.	Define covariance of X, Y and coefficient of correlation between X and Y.	2	K1	CO5
22.	If the function $f(x, y) = c(1 - x)(1 - y), 0 < x < 1, 0 < y < 1$ is to be the joint probability density function of a two-dimensional continuous random variable, then find the value of c.	2	K2	CO5

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) Find the general integral of $(mz - ny)p + (nx - lz)q = ly - mx$. *11 K3 CO1*

OR

b) Solve $\left[D^3 - 7DD'^2 - 6D'^3\right] z = sin(x + 2y) + e^{2x+y}$. *11 K3 CO1*

24. a) A string is stretched and fastened to two end points l apart. Motion is started by displacing the string in to the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point of the string. *11 K3 CO2*

OR

b) A rod 30cm long has its ends A and B kept at $20°C$ and $80°C$ respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0°C$ and kept so. Find the resulting temperature function $u(x, t)$. *11 K3 CO2*

25. a) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a > 0 \end{cases}$. Hence deduce that $\int_0^\infty \frac{sint - tcost}{t^3} dt = \frac{\pi}{4}$. *11 K3 CO3*

OR

b) Find the Fourier sine transform of $f(x) = e^{-ax}$, and $g(x) = e^{-bx}, a, b > 0$ and hence evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$. *11 K3 CO3*

26. a) A random variable X has the following probability distribution. *11 K3 CO4*

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find:

(i) The value of K.

(ii) $P(1.5 < X < 4.5 / X > 2)$.

(iii) Evaluate $P(X < 6)$, $P(X \ge 6)$, and $P(0 < X < 5)$.

OR

	b)	In a bolt factory, machines A,B,C manufacture 25%, 35%, 40% of the total output respectively. Out of their outputs 5, 4, 2 percent respectively are defective bolts. A bolt is drawn at random from the output and is found to be defective. Find the probability that it was manufactured by machine A?	*11*	*K3*	*CO4*
27.	a)	The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y)$, $x = 0, 1, 2;\ y = 1, 2, 3$. Find the marginal probability distributions. Also find the probability distribution of $X + Y$ and the conditional distribution of X given $Y = 1$.	*11*	*K3*	*CO5*
		OR			
	b)	If $X_1, X_2, \ldots\ldots, X_n$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P[120 \leq S_n \leq 160]$, where $S_n = X_1 + X_2 + \ldots + X_n$ and $n = 75$.	*11*	*K3*	*CO5*
28.	a) (i)	Bus arrives at a stop at 15 minutes intervals starting at 7 A.M. If a passenger's arrival time to the station is uniformly distributed between 7 A.M. and 7.30 A.M., find the probability that he has to wait for the bus for i) less than 5 minutes ii) more than 10 minutes.	*6*	*K3*	*CO4*
	(ii)	The lines of regression of a bivariate population are: $8x - 10y + 66 = 0$ and $40x - 18y = 214$ and Variance of $X = 9$. Find (i) The mean values of X and Y. (ii) The correlation coefficient between X and Y.	*5*	*K3*	*CO5*
		OR			
	b) (i)	Find the moment generating function of Poisson distribution. Hence find the mean of the distribution.	*6*	*K3*	*CO4*
	(ii)	If $f(x,y) = \begin{cases} 2 - x - y, & 0 \leq x, y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$ is the joint pdf of the random variables X and Y, find the mean of X and Y.	*5*	*K3*	*CO5*