Reg. No.																

Question Paper Code	13712

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2025

Third Semester

Computer Science and Engineering (Cyber Security)

20BSMA309 – NUMBER THEORY

Regulations - 2020

Duration: 3 Hours — Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

		Marks	K – Level	CO
1.	Express $(101011111)_2$ in base 10. (a) 351 (b) 565 (c) 769 (d) 234	1	K2	CO1
2.	Find the GCD of 504 and 540. (a) 56 (b) 55 (c) 36 (d) 34	1	K2	CO1
3.	If $ax \equiv ay (mod\ m)$ and $(a, m) = 1$, then $x \equiv \ldots (mod\ m)$ (a) y (b) $x + y$ (c) $x - y$ (d) xy	1	K1	CO2
4.	$x^2 + 1 \equiv 0 (mod\ 5)$ has …. Solutions. (a) one (b) No (c) Two (d) Three	1	K2	CO2
5.	Suppose that Q and Q' are odd and positive, then $\left(\frac{P}{Q}\right)\left(\frac{P}{Q'}\right) = \ldots.$ (a). $\left(\frac{P}{QQ'}\right)$ (b) $\left(\frac{P}{Q+Q'}\right)$ (c) $\left(\frac{P}{Q-Q'}\right)$ (d) $\left(\frac{P^2}{QQ'}\right)$	1	K1	CO3
6.	Find the value of $\left\lceil \frac{1000}{7} \right\rceil = \ldots..$ (a) 156 (b) 143 (c)142 (d) 234	1	K2	CO3
7.	The general form of a Linear Diophantine equation in two variables x and y is …… (a) $ax + by = c$ (b) $ax - by = b$ (c) $ax - by = c$ (d) $ax + by = b$	1	K1	CO4
8.	Every positive integer can be expressed as the sum of a fixed number of n^{th} powers. This statement is denoted by ……….. (a) Fermat's Theorem (b) Wilson's Theorem (c) Waring's Problem (d) Euler's Theorem	1	K1	CO4
9.	A positive integer p is a prime if and only if $\phi(p) = \ldots\ldots\ldots\ldots$ (a) $p + 1$ (b) $p - 1$ (c) p (d) p^3	1	K1	CO5
10.	Compute $\tau(81)$. (a) 4 (b) 5 (c) 7 (d) 6	1	K2	CO5

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

11.	Determine whether 1601 is a prime number.	2	K2	CO1
12.	Determine the positive integer a, if $[a, a + 1] = 132$.	2	K2	CO1
13.	Using the number pattern $1^2 - 0^2 = 1$ $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ … … … … … .. Make a conjecture about row n and prove the conjecture.	2	K2	CO1
14.	How many solutions are there to each of the following congruences? $15x \equiv 24 (mod\ 35)$.	2	K2	CO2
15.	Solve $x^2 + x + 1 \equiv 0 (mod 7)$.	2	K2	CO2

16.		Determine the number of incongruent solutions $48x \equiv 144 \ (mod\ 84)$.	2	K2	CO2
17.		Find all primes p such that $\left(\frac{10}{p}\right) = 1$.	2	K2	CO3
18.		Define Jacobi symbol.	2	K1	CO3
19.		Examine whether the Linear Diophantine equation $12x + 16y = 18$ is solvable or not.	2	K2	CO4
20.		Express the number 353 as a sum of fourth powers.	2	K2	CO4
21.		Show that $p = 11, (p-1)! = 10!$ is divisible by 11, using Wilson's Theorem.	2	K2	CO5
22.		Compute $\tau(6120)$.	2	K2	CO5

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23.	a)	State and Prove Fundamental theorem of Arithmetic.	11	K3	CO1
		OR			
	b)	Apply Euclidean algorithm to express the gcd of 1976 and 1776 as a linear combination of themselves.	11	K3	CO1
24.	a)	Solve $x^2 + x + 47 \equiv 0 \ (mod\ 7^3)$.	11	K3	CO2
		OR			
	b)	Find the least positive integer that leaves remainder 3 when divided by 7, 4 when divided by 9 and 8 when divided by 11.	11	K3	CO2
25.	a)	Let p be an odd prime, and $a\&b$ be any integers with p does not divides ab. Then, prove that (i) If $a \equiv b \ (modp)$, then $\left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$. (ii) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$. (iii) $(a^2/p) = 1$.	11	K3	CO3
		OR			
	b)	State and prove Mobius inversion formula.	11	K3	CO3
26.	a)	Twenty-three weary travellers entered the outskirts of a lush and beautiful forest. They found 63 equal heaps of plantains and seven single fruits, and divided them equally. Find the number of fruits in each heap. Find the Linear Diophantine equation for the problem	11	K3	CO4
		OR			
	b)	Prove that Every positive integer n can be expressed as the sum of the four squares, $n = x_1^2 + x_2^2 + x_3^2 + x_4^2$, where x_i arenon-negative integers.	11	K3	CO4
27.	a)	State and prove Fermat's Little's Theorem.	11	K3	CO5
		OR			
	b)	Using Euler's theorem find the remainder when 245^{1040} is divided by 18.	11	K3	CO5
28.	a) i)	Determine the general solution of the Linear Diophantine equation $6x + 8y + 12z = 10$.	6	K3	CO4
	ii)	Let p be a prime and n any positive integer, Prove that $\frac{(np!)}{n!\,p^n} \equiv (-1)^n (\text{mod}\ p)$	5	K3	CO5
		OR			
	b) i)	Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares.	6	K3	CO4
	ii)	Let p be any prime and e any positive integer. Then$\tau(p^e) = e + 1$ and $\sigma(p^e) = \frac{p^{e+1} - 1}{p - 1}$.	5	K3	CO5