Reg. No.																

Question Paper Code	13576

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2025

Fourth Semester

Electronics and Communication Engineering

(Common to Computer and Communication Engineering)

20BSMA401 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

Regulations - 2020

(Use of Statistical Table is permitted)

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

		Marks	*K – Level*	*CO*
1.	If X and Y are independent random variables with variance 2 and 3, then the variance of $3X + 4Y$ is (a) 68 (b) 66 (c) 64 (d) 62	*1*	*K2*	*CO1*
2.	A random variable X is uniformly distributed between 3 and 15, then the variance of X is (a) 8 (b) 10 (c) 12 (d) 14	*1*	*K2*	*CO1*
3.	The value of $\sum\sum p(x_i, y_j)$ is equal to (a) 1 (b) 0 (c) ∞ (d) 2	*1*	*K1*	*CO2*
4.	Two regression lines are $5x - y = 22, 64x - 45y = 24$, then the mean of X is (a) 2 (b) 4 (c) 6 (d) 8	*1*	*K2*	*CO2*
5.	The cross correlation $R_{XY}(\tau) =$ (a) $R_{YX}(\tau)$ (b) $R_{XY}(\tau)$ (c) $R_{XY}(-\tau)$ (d) $R_{YX}(-\tau)$	*1*	*K1*	*CO3*
6.	The average power of the process of the auto correlation function $R_{xx}(\tau) = \frac{1}{2}e^{-2\lvert\tau\rvert} - \tau e^{-2\lvert\tau\rvert}$ is (a) $\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$	*1*	*K2*	*CO3*
7.	The Bernoulli random variable is used when an experiment has (a) only one outcome (b) only two outcomes (c) n outcomes (d) infinite number of outcomes	*1*	*K1*	*CO4*
8.	The Poisson process is a (a) WSS process (b) Ergodic process (c) Markov chain (d) Markov process	*1*	*K1*	*CO4*
9.	A system which is linear and time invariant is known as (a) Linear invariant system. (b) Time invariant system. (c) Time Linear invariant system. (d) Linear Time invariant system.	*1*	*K1*	*CO5*
10.	The Power Spectral $S_{xx}(\omega) =$ (a) $S_{xy}(\omega)$ (b) $S_{yx}(\omega)$ (c) $S_{xy}(-\omega)$ (d) $S_{yx}(-\omega)$	*1*	*K1*	*CO5*

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

		Marks	*K – Level*	*CO*
11.	Define the discrete random variable with an example.	*2*	*K1*	*CO1*
12.	State any two properties of moment generating functions.	*2*	*K1*	*CO1*
13.	The length of time a person speaks over phone follows exponential distribution with mean 1/6. What is the probability that the person will talk for more than 8 minutes.	*2*	*K2*	*CO1*
14.	The joint PDF of X and Y is given by $p_{X,Y}(x,y) = \begin{cases} kxy, & x = 1,2,3; y = 1,2,3 \\ 0, & otherwise \end{cases}$. Determine the value of the constant k.	*2*	*K2*	*CO2*

15.	Distinguish between the correlation and the regression.	2	K2	CO2
16.	State the Central Limit Theorem.	2	K1	CO2
17.	State any two properties of an auto correlation function.	2	K1	CO3
18.	Define an Ergodic Process.	2	K1	CO3
19.	State any two properties of the Gaussian process.	2	K1	CO4
20.	What is Markov process?	2	K1	CO4
21.	Prove that $Y(t) = 2X(t)$ is linear.	2	K2	CO5
22.	Given the power spectral density of a stationary process $S_{xx}(\omega) = \frac{1}{4+\omega^2}$. Find the average power of the process.	2	K2	CO5

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23\. a) The probability density function of a random variable X is given by $f_X(x) = \begin{cases} x, 0 < x < 1 \\ k(2-x), 1 \le x \le 2 \\ 0, Otherwise \end{cases}$ *11 K3 CO1*

Find the value of 'k'

Find $P(0.2 < x < 1.2)$

What is $P[0.5 < x < 1.5/x \ge 1]$

OR

b) The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 70 with a standard deviation of Rs. 5. Estimate the number of workers whose weekly wages will be (i) Between Rs. 69 and Rs. 72, (ii) Less than Rs. 69, (iii) More than Rs. 72 *11 K3 CO1*

24\. a) Two random variables X and Y have the joint probability density function given by $f_{XY}(x,y) = \begin{cases} 25e^{-5y}, & 0 < x < 0.2, y > 0 \\ 0, & otherwise \end{cases}$. (i) Find the marginal PDF of X and Y, (ii) What is the covariance of X and Y? *11 K3 CO2*

OR

b) Two random variables X and Y have the following joint probability density function $f(x,y) = \begin{cases} x+y, 0 \le x \le 1; 0 \le y \le 1 \\ 0, \quad otherwise \end{cases}$. Find the probability density function of the random variable $U = XY$. *11 K3 CO2*

25\. a) Determine the mean and variance of the process, given that the auto correlation functions $R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$. *11 K3 CO3*

OR

b) Find the power spectral density of the random process whose auto correlation function is $R_{xx}(\tau) = \begin{cases} 1-|\tau|, for \ |\tau| \le 1 \\ 0, otherwise \end{cases}$ *11 K3 CO3*

26\. a) The probability matrix of a Markov chain $\{X_n\}, n = 1,2,3\ldots.$ having three states 0, 1 and 2 is $A = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution is $P^{(0)} = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$. Find *11 K3 CO4*

(i) $P(X_3 = 2/X_2 = 1)$ (ii) $P(X_2 = 2, X_1 = 1, X_0 = 2)$

(iii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$ (iv) $P(X_2 = 2)$

OR

	b)	In reliance super market; there are two brands of rice A and B. A customer buys brand A with probability 0.7 if his last purchase was A and buys brand B with probability 0.4 if his last purchase was B. Assuming markov chain model, obtain (i) one step tpm P, (ii) 2 – step tpmP^2, (iii) the stationary distribution. Hence highlight the proportion of customers who would buy brand A and brand B in the long run.	*11*	*K3*	*CO4*
27.	a)	A random process $X(t)$ is input to a linear system whose impulse function is$h(t) = 2e^{-t}; t \geq 0$. The auto correlation function of the process is $R_{xx}(\tau) = e^{-2\|\tau\|}$. Find the power spectral density of the output process Y(t).	*11*	*K3*	*CO5*
		OR			
	b)	If $\{X(t)\}$ is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$, then prove that $(i)\ R_{XY}(\tau) = R_{XX}(\tau) * h(\tau)$ $(ii)\ R_{YY}(\tau) = R_{XY}(\tau) * h(-\tau)$ $(iii)\ S_{XY}(\omega) = S_{XX}(\omega)H(\omega)$, if $*$ denotes convolution operation.	*11*	*K3*	*CO5*
28.	a) i)	If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16e^{-\|t_1 - t_2\|}$, find the probability that $X(10) \leq 8$.	*6*	*K3*	*CO4*
	ii)	Write the properties of Linear Systems with Random Inputs.	*5*	*K3*	*CO5*
		OR			
	b) i)	A fisherman catches a fish at a Poisson rate of 2 per hour from a large lake with lots of fish. If he starts fishing at 10 am. What is the probability that he catches one fish by 10.30 am and three fishes by noon?	*6*	*K3*	*CO4*
	ii)	A linear time in variant system has an impulse response $h(t) = e^{-\beta\tau}u(\tau)$. Find the output autocorrelation function $R_{YY}(\tau)$ corresponding to an input $X(t)$.	*5*	*K3*	*CO5*