Reg. No.																

Question Paper Code	13577

B.E. / B.Tech./ M.Tech - DEGREE EXAMINATIONS, APRIL / MAY 2025

Fourth Semester

Computer Science and Engineering

(Common to Information Technology, Computer Science and Engineering (Cyber Security) & M.Tech - Computer Science and Engineering (5 Years Integrated))

20BSMA402 - PROBABILITY AND QUEUEING THEORY

Regulations - 2020

(Use of Data Books, Tables is permitted)

Duration: 3 Hours　　　　Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
1.	The Mean, Median and Mode coincide in (a) Exponential Distribution (b) Normal Distribution. (c) Poisson Distribution. (d) Binomial Distribution.	1	K1	CO1
2.	A coin is tossed twice. The sample space is given by (a) $S = \{H, T\}$ (b) $S = \{HH, TT, TH, HT\}$ (c) $S = \{1,2\}$ (d) $S = \{1,2,3,4,5,6\}$	1	K2	CO1
3.	If the random variables X and Y are independent then Cov(X,Y) = (a) 0 (b) 1 (c) $E(XY)$ (d) $E(X)E(Y)$	1	K1	CO2
4.	Two regression lines coincide when (a) $r = 0$ (b) $r = 2$ (c) $r = 3$ (d) $r = \pm 1$	1	K1	CO2
5.	The autocorrelation function of a random process is a function of: (a) Time only (b) Time and amplitude (c) Time difference (τ) between two points (d) Amplitude difference between two points	1	K1	CO3
6.	For a Poisson process with parameter λ the time between two successive events follows which distribution? (a) Uniform distribution (b) Exponential distribution (c) Normal distribution (d) Gamma distribution	1	K1	CO3
7.	What is the utilization factor (ρ) in an M/M/1 queue? (a) λ/μ (b) μ/λ (c) λ + μ (d) μ – λ	1	K1	CO4
8.	Which of the following is true about the M/M/c queue? (a) It has only one server (b) It has multiple servers with Poisson arrivals and exponential service times (c) It can only serve one customer at a time (d) It cannot handle more than c customers in the system	1	K1	CO4
9.	In (M/G/1) model, the formula for the average waiting time in the queue is (a) $\frac{\lambda^2\sigma^2+\rho\mu^2}{2[1-\rho]}$ (b) $\frac{\lambda^2\sigma^2-\rho^2}{\lambda[1-\rho]}$ (c) $\frac{\lambda^2\sigma^2+\rho^2}{2[1-\rho]}$ (d) $\frac{\lambda^2\sigma^2+\rho^2}{2\lambda[1-\rho]}$	1	K1	CO5
10.	Using Kendall's notation to classify a queue configuration, the term M indicates which probability distribution? (a) Normal (b) Exponential (c) Gamma (d) Poisson	1	K1	CO5

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

No.	Question	Marks	K – Level	CO
11.	A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1 + x)$. Find $P[X < 4]$.	2	K2	CO1
12.	State and prove Memoryless property of the exponential distribution.	2	K2	CO1
13.	A normal distribution has mean $\mu = 20$ and standard deviation $\sigma = 10$. Find $P(15 \le X \le 40)$.	2	K2	CO1

14. State the formula for angles between two regression lines. *2 K1 CO2*
15. State Central limit theorem. *2 K1 CO2*
16. If X and Y are independent random variables with variances 2 and 3. Find the variance of $3X + 4Y$. *2 K2 CO2*
17. State Chapman - Kolmogorov theorem. *2 K1 CO3*
18. Check whether the given transition probability matrix is regular or not. *2 K2 CO3*

$$\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

19. If there are two servers in an infinite capacity Poisson queue system with $\lambda = 10$ per min $\mu = 15$ per minutes, what is the percentage of idle time for each server? *2 K2 CO4*
20. What are the basic characteristics of a queuing system? *2 K1 CO4*
21. Define 'Bottle neck' of the system in queue networks. *2 K1 CO5*
22. State Jackson's theorem for an open network. *2 K1 CO5*

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) A random variable X has the following probability function: *11 K3 CO1*

$X = x$	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find

(i) k

(ii) Evaluate $P(X < 6), P(X \geq 6)$ and $P(0 < X < 5)$

(iii) If $P(X \leq k) > \frac{1}{2}$, find the least value of k.

(iv) Find the cumulative distribution function of X.

OR

b) In a bolt manufacturing factory, machines A, B, C manufacture 25%, 35% and 40% of the total output respectively. Of their output 5%, 4% and 2% are defective bolts respectively. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B, C? *11 K3 CO1*

24. a) The joint probability mass function of (X, Y) is given by $P(x, y) = K(2x + 3y$, $x=0, 1, 2; y=1, 2, 3$. Find all the marginal and conditional probability distributions. *11 K3 CO2*

OR

b) If the joint probability density function of (X, Y) is given by $f(x, y) = x + y$, $0 \leq x, y \leq 1$, find the probability density function of $U = XY$. *11 K3 CO2*

25. a) The process $\{X(t)\}$ whose probability distribution under certain condition is given by $P[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3, \ldots \\ \frac{at}{1+at}, & n = 0 \end{cases}$ *11 K3 CO3*

Show that $\{X(t)\}$ is not stationary.

OR

b) A man either drives a car or catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run. *11 K3 CO3*

26.	a)	Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. (i) Find the effective arrival rate at the clinic. (ii) What is the probability that an arriving patient will not wait? (iii)What is the expected waiting time until a patient is discharged from the clinic?	*11*	*K3*	*CO4*
		OR			
	b)	A two-person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min. in the barber's chair. Compute $P_0, P_1, P_7, E\left(N_q\right)$.	*11*	*K3*	*CO4*
27.	a)	Derive Pollaczek – Khinchine formula.	*11*	*K3*	*CO5*
		OR			
	b)	An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes then, determine $L_s, L_q, W_s, and\ W_q$.	*11*	*K3*	*CO5*
28.	a) (i)	Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is exponential with mean 10 minutes. Determine L_s.	*6*	*K3*	*CO4*
	(ii)	A one-man barbershop takes 25 minutes to complete one haircut. If customers arrive at the barbershop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop?	*5*	*K3*	*CO5*
		OR			
	b) (i)	A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min. and car arrive for service in a Poisson process at the rate of 30 cars per hour. What is the probability that an arrival would have to wait in line?	*6*	*K3*	*CO4*
	(ii)	Consider a queuing system where arrivals are according to a Poisson distribution with mean 5/hour. Find the expected waiting time in the system if the service time distribution is Uniform from $t = 5$ min to $t = 15$ min.	*5*	*K3*	*CO5*